Introductory Control Systems
Block Diagrams and Transfer Functions – Example

Problem:

The angular position of a rotating shaft is controlled with a torque actuator with a proportional-integral (PI) controller. The desired shaft angle is $\theta_d(t)$, the actual shaft angle is $\theta(t)$, the angle error is $e(t)$, the input voltage to the actuator is $v(t)$, and the actuator torque is $m(t)$. The equations that govern the individual processes are as follows.

$$\boxed{e(t)=\theta_d(t)-\theta(t)} \quad \boxed{v(t)=2e(t)+8\int_0^t e(t)dt} \quad \boxed{\dot{m}+3m=v(t)} \quad \boxed{\ddot{\theta}+7\dot{\theta}=4m(t)}$$

In the last two equations, the ***over-dot*** represents the ***time derivative***.

Find:

a) transfer functions $\frac{V}{E}(s)$, $\frac{M}{V}(s)$, and $\frac{\theta}{M}(s)$.

b) Sketch the block diagram of the closed-loop system. Label all signals and transfer functions.

c) Using block diagram reduction, fid the closed-loop system transfer function $\frac{\theta}{\theta_d}(s)$.

Solution:

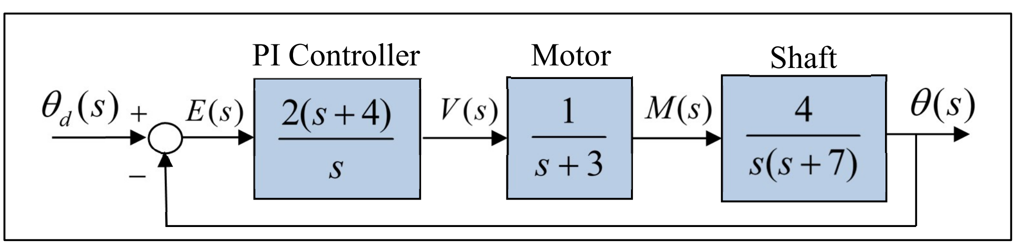
a) Applying Laplace transforms to the model equations gives

$$V(s)=2E(s)+8\frac{E(s)}{s}=\left(2+\tfrac{8}{s}\right)E(s)=\left(\frac{2(s+4)}{s}\right)E(s) \quad \Rightarrow \boxed{\frac{V}{E}(s)=\frac{2(s+4)}{s}}$$

$$sM(s)+3M(s)=V(s) \quad \Rightarrow \boxed{\frac{M}{V}(s)=\frac{1}{s+3}}$$

$$s^2\theta(s)+7s\,\theta(s)=4M(s) \quad \Rightarrow \boxed{\frac{\theta}{M}(s)=\frac{4}{s(s+7)}}$$

b)



c) Using block diagram reduction, with $G = N_G / D_G$ and $H = 1$

$$\frac{\theta}{\theta_d}(s) = \frac{G}{1+GH} = \frac{N_G/D_G}{1+N_G/D_G} = \frac{N_G}{D_G + N_G} = \frac{8(s+4)}{s^2(s+3)(s+7)+8(s+4)}$$

$$\Rightarrow \boxed{\frac{\theta}{\theta_d}(s) = \frac{8(s+4)}{s^4+10s^3+21s^2+8s+32}}$$