Introductory Control Systems
Block Diagram Transformations

Reference: R.C. Dorf and R.H. Bishop, *Modern Control Systems*, 11th Ed., Pearson/Prentice-Hall, 2008.

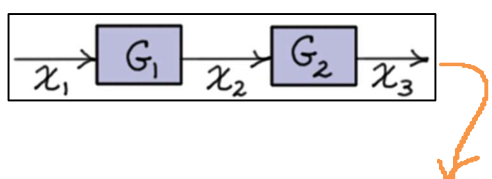
Complex block diagrams can be ***transformed*** into ***simpler equivalent diagrams*** using basic ***block diagram transformations***. These transformations are ***derived*** by simply manipulating the ***algebraic equations*** associated with the diagram. The list of transformations derived below is ***not meant*** to be ***all inclusive***, but rather to encourage the reader to begin to ***understand*** block diagram transformations by ***reading*** the ***details*** of the block diagram.

Motivation: It is ***not intended*** here that the analyst become proficient at reducing large complex block diagrams, but rather should be able to ***read*** the ***details*** provided by the block diagram. If the ***details*** of the block diagram ***truly reflect*** the ***function*** of the system, then ***understanding*** the ***block diagram*** is the same as ***understanding*** the ***operation*** of the ***system*** itself. ***Automated procedures*** are available to assist the analyst in the reduction of complex block diagrams.

Block Diagram Transformations

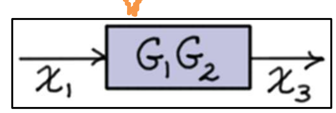
In the transformations shown below, simple block diagrams are reduced to simpler forms by using the ***block diagram algebra*** associated with the diagram. In each case, the first figure is the starting diagram, and the second figure is the reduced "***equivalent***" diagram.

1. Combining blocks in a series:

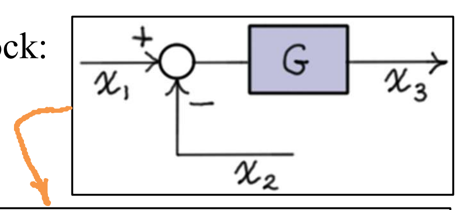


Algebra:

$x_3 = G_2 x_2 = G_2 G_1 x_1$ $\Rightarrow$ $\dfrac{x_3}{x_1} = G_2 G_1 = G_1 G_2$ $\Rightarrow$

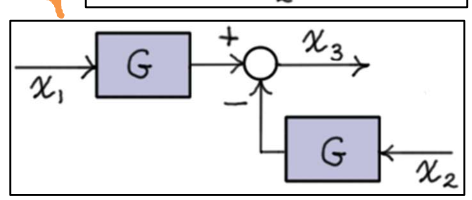


2. Moving a summing point behind a block:

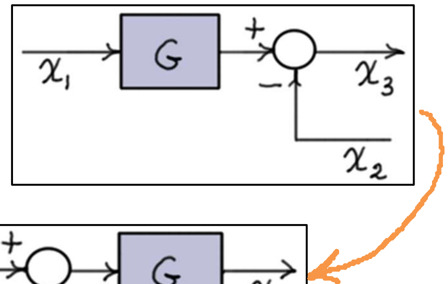


Algebra:

$x_3 = G(x_1 - x_2) = G\,x_1 - G\,x_2$ $\Rightarrow$



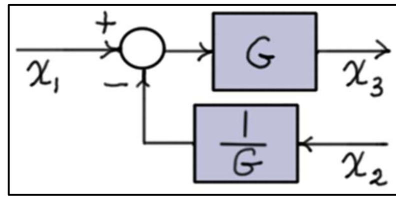
3. Moving a summing point in front of a block:



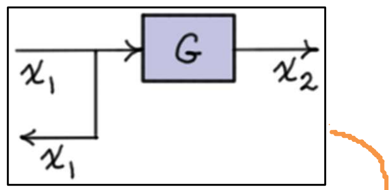
Algebra:

$$x_3 = G\,x_1 - x_2 = G\left(x_1 - \left(\frac{1}{G}\right) x_2 \right)$$

$\Rightarrow$

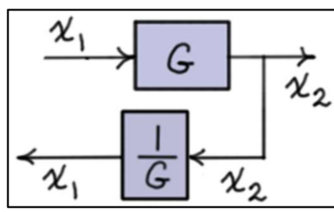


4. Moving a pick-off point behind a block:

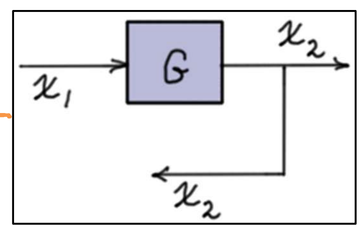


Algebra:

$$x_2 = G\,x_1$$ $\Rightarrow$ $$x_1 = \left(\frac{1}{G}\right) x_2$$ $\Rightarrow$

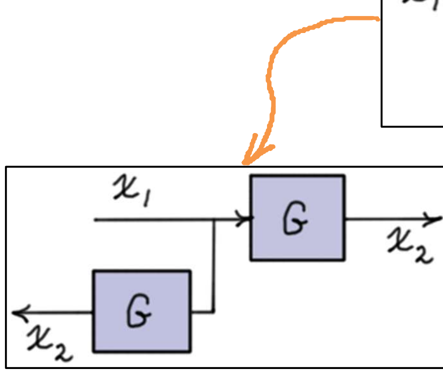


5. Moving a pick-off point in front of a block:

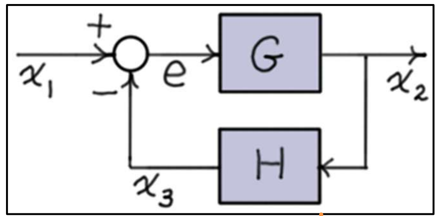


Algebra:

$$x_2 = G\,x_1$$ $\Rightarrow$



6. Collapsing a feedback loop:

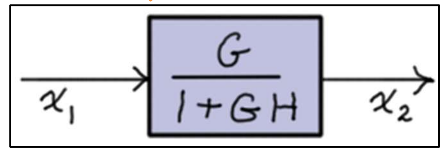


Algebra:

$$x_2 = G\,e = G\left(x_1 - x_3\right) = G\,x_1 - G\,x_3 = G\,x_1 - G\,H\,x_2$$

$$\Rightarrow \left(1 + G\,H\right)x_2 = G\,x_1 \quad \Rightarrow \boxed{\frac{x_2}{x_1} = \frac{G}{1 + G\,H}}$$

$\Rightarrow$



As shown, the system is said to have "***negative feedback***", because the signal x_3 is ***negated*** at the summing block. If the system has "***positive feedback***", then it is easy to show that the transfer function becomes

$$\boxed{\frac{x_2}{x_1} = \frac{G}{1 - G\,H}}$$ (for positive feedback)

Notes:

- In each transformation, signals may be ***modified*** or even ***eliminated*** from the system. So, the transformed system is ***not*** "***identical***" to the original. The transformed system is "***equivalent***" to the original system in that it has the same input and output signals. Of course, the ***reduced system*** must have the ***same transfer function*** as the ***original system***.
- When transforming a block diagram, it is important ***not*** to ***change*** or ***eliminate*** signals required by other portions of the diagram. These types of changes will generally render the transformed diagram to be ***not equivalent*** to the original.
- When reducing systems that have ***multiple closed loops***, the original system should be transformed into one whose closed loops are ***nested***. This is accomplished using transformations like items (1) through (5) above.
- Once the system is in a nested form, it can be reduced by collapsing the ***inner-most loop*** first and then collapsing ***each successive*** inner-most loop until the outer-most loop is collapsed.

- For the simple closed loop system of item (6), the following ***terminology*** is often used:
  - Closed loop transfer function: $\frac{x_2}{x_1} = \frac{G}{1 \pm GH}$
  - Forward path transfer function: $\frac{x_2}{e} = G$
  - Feedback path transfer function: $\frac{x_3}{x_2} = H$
  - Loop transfer function: $\frac{x_3}{e} = GH$