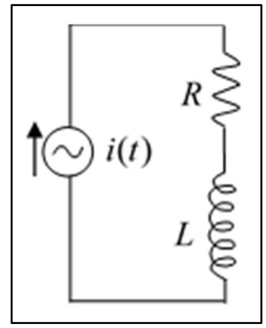
Elementary Engineering Mathematics
Application of Sine and Cosine Functions in Electrical Engineering



Given: A current source $i(t) = 2\sin(120\pi t)$ is applied to an RL series circuit with $R = 100$ (ohms) and $L = 100$ (mh). The voltage drop across the resistor is $\boxed{V_R = Z_R I}$, the voltage drop across the inductor is $\boxed{V_L = Z_L I}$, and the total voltage drop across both elements is $\boxed{V = V_R + V_L}$.

Find: (a) $v(t)$ the voltage as a function of time. (b) $v(t)$ as a single, phase-shifted ***sine wave***. (c) $v(t)$ as a single, phase-shifted ***cosine wave***.

Solution:

a) $\boxed{V_R = Z_R I = \left(100\angle 0^\circ\right)\left(2\angle -90^\circ\right) = 200\angle -90^\circ}$

or $\boxed{v_R(t) = 200\cos(120\pi t - 90) = 200\sin(120\pi t)}$

$\boxed{V_L = Z_L I = \left((120\pi(0.1))\angle 90^\circ\right)\left(2\angle -90^\circ\right) \approx 75.4\angle 0^\circ}$ or $\boxed{v_L(t) \approx 75.4\cos(120\pi t)}$

So,

$\boxed{v(t) \approx 200\sin(120\pi t) + 75.4\cos(120\pi t)}$

b) $\boxed{v(t) = M\sin(120\pi t + \phi)}$

$\boxed{M = \sqrt{200^2 + 75.4^2} \approx 214 \text{ (volts)}}$ $\boxed{\phi \approx \tan^{-1}\left(75.4/200\right) \approx 20.66^\circ \approx 0.3605 \text{ (rad)}}$

c) $\boxed{v(t) = M\cos(120\pi t + \phi)}$

$\boxed{M = \sqrt{200^2 + 75.4^2} \approx 214 \text{ (volts)}}$ $\boxed{\phi \approx \tan^{-1}\left(-200/75.4\right) \approx -69.34^\circ \approx -1.21 \text{ (rad)}}$

Alternate Solution:

$\boxed{V_R = Z_R I = 200\angle -90^\circ = -j200}$ and $\boxed{V_L = Z_L I \approx 75.4\angle 0^\circ \approx 75.4}$

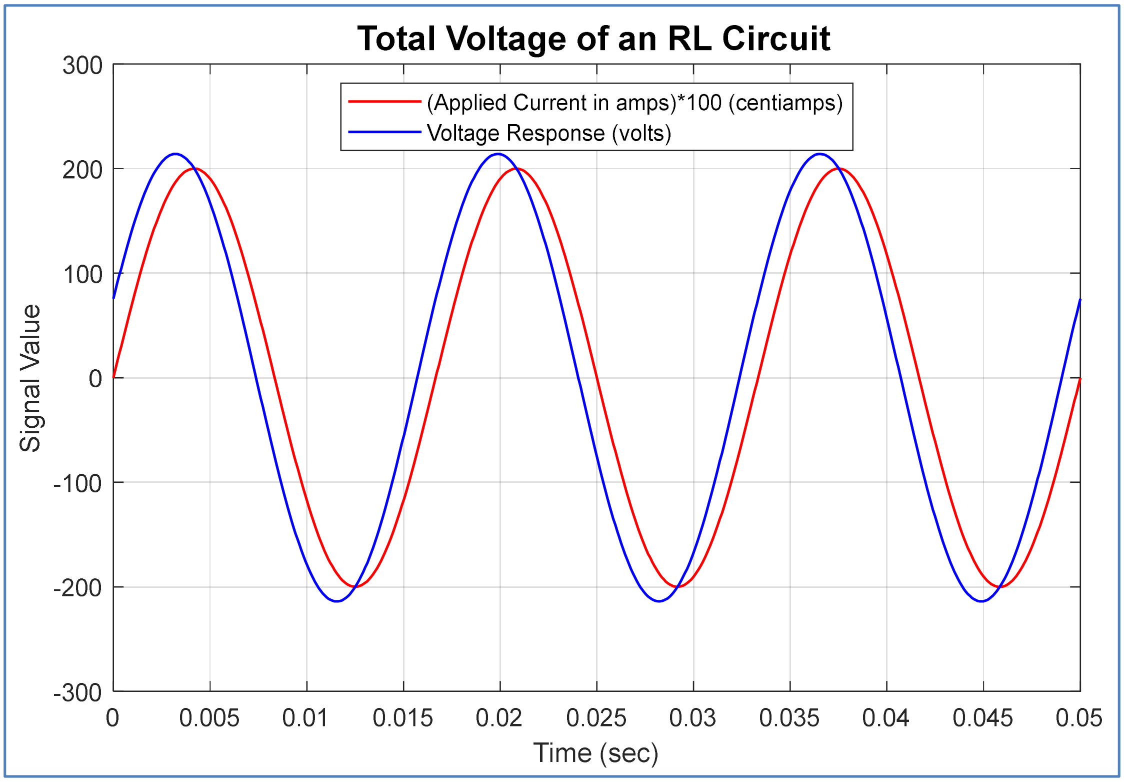
So,

$$\boxed{V = 75.4 - j200 = \left(\sqrt{75.4^2 + 200^2}\right)\angle \tan^{-1}\left(-200/75.4\right) \approx 214\angle\left(-1.21\right) \text{ (volts)}}$$

Or,

$$\begin{aligned} v(t) &= 214\cos\left(120\pi t - 1.21\right) = 214\sin\left(120\pi t - 1.21 + \pi/2\right) \\ &= 214\sin\left(120\pi t + 0.3605\right) \text{ (volts)} \end{aligned}$$

(See plot below)



Total Voltage of an RL Circuit
(Applied Current in amps)*100 (centiamps)
Voltage Response (volts)
Signal Value
300
200
100
0
-100
-200
-300
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04
0.045
0.05
Time (sec)