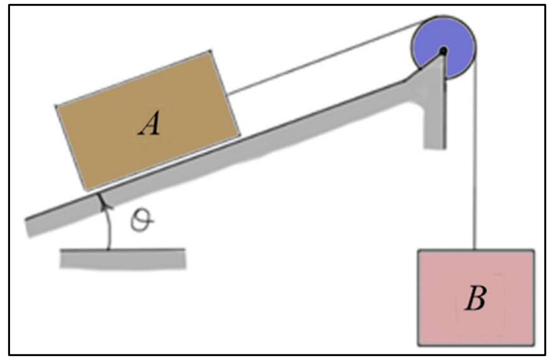
Elementary Dynamics Example #20: (Work & Energy)



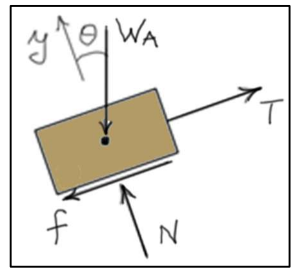
Given: $W_A = 100$ (lb), $W_B = 110$ (lb),
$\mu_k = 0.3$, $\theta = 20$ (deg)
system is ***released from rest*** and *B* moves down

Find: velocities of *A* and *B* after moving a distance $d = 2$ (ft)

Solution:

Newton's 2nd Law:

Free body diagram



$$A: \boxed{\sum F_y = N - W_A\cos(\theta) = 0} \Rightarrow \boxed{f = \mu_k N = \mu_k W_A \cos(\theta)}$$

Work & Energy: (applied to the system)

$$\boxed{\underbrace{K_1}_{\text{zero}} + U_{1\to 2} = K_2} \text{ with } \boxed{U_{1\to 2} = \left(U_{1\to 2}\right)_{\text{friction}} + \left(U_{1\to 2}\right)_{W_A} + \left(U_{1\to 2}\right)_{W_B}}$$

where

$$\left.\begin{aligned} \left(U_{1\to 2}\right)_{\text{friction}} &= -f\,d = -\mu_k W_A \cos(\theta)\, d \\ \left(U_{1\to 2}\right)_{W_A} &= -W_A\, d\sin(\theta) = V_1 - V_2 \\ \left(U_{1\to 2}\right)_{W_B} &= W_B\, d = V_1 - V_2 \end{aligned}\right\} \boxed{U_{1\to 2} = 95.2144 \text{ (ft-lb)}}$$

Note that because $U_{1\to 2} > 0$, it confirms that *B* ***moves down***.

$$\boxed{K_2 = \tfrac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 + \tfrac{1}{2}\left(\frac{W_A}{g}\right)v_A^2 = \tfrac{1}{2}\left(\frac{W_A + W_B}{g}\right)v^2} \text{ (blocks have the same velocity)}$$

Substituting into the ***work & energy equation***

$$\tfrac{1}{2}\left(\frac{W_A + W_B}{g}\right)v^2 = U_{1\to 2} \Rightarrow \boxed{v = \sqrt{\frac{2gU_{1\to 2}}{W_A + W_B}} \approx 5.40 \text{ (ft/s)}}$$

Observations:

1. We treated the two blocks and the connecting cable as a ***single system***, and consequently had a ***single work & energy equation***. The ***net*** work of the cable tension on the system is ***zero***, so it ***does not contribute*** to this equation. The work done by the cable tension on block *A* is positive (causing it to move up the plane), and the work it does on block *B* is of the same magnitude but negative (slowing its downward motion).
2. If we treated the ***two blocks separately***, we could write a ***work & energy equation for each block***. The cable tension does ***nonzero*** work on each block, so the cable tension is an ***unknown*** in these two equations.