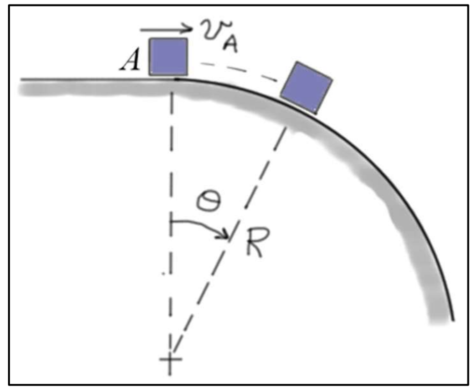
Elementary Dynamics Example #21: (Work & Energy)

Given: $W_{box} = W = 5$ (lb), $R = 2$ (ft), $v_A = 4$ (ft/s), surface is smooth (no friction)

Find: $\hat{\theta}$ the angle where the box begins to leave the surface



Solution: using work & energy and Newton's law

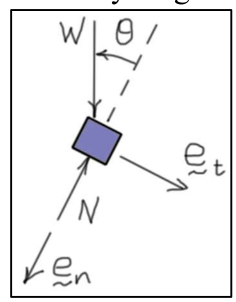
Newton's Law:

$$\swarrow \sum F_n = W\cos(\theta) - N = \left(\frac{W}{g}\right)\left(\frac{v^2}{R}\right)$$

At the point where the box leaves the surface, $N = 0$ and $\theta = \hat{\theta}$, so

$$W\cos(\hat{\theta}) = \left(\frac{W}{g}\right)\left(\frac{v^2}{R}\right) \Rightarrow v^2 = g\,R\cos(\hat{\theta})$$

Free body diagram



Work & Energy: (applied to the box)

$$K_1 + U_{1\to 2} = K_2 \quad \text{with} \quad U_{1\to 2} = \left(U_{1\to 2}\right)_W = W\left(R - R\cos(\hat{\theta})\right) = V_1 - V_2$$

Substituting into the work and energy equation:

$$\tfrac{1}{2}\left(\frac{W}{g}\right)v_A^2 + W\left(R - R\cos(\hat{\theta})\right) = \tfrac{1}{2}\left(\frac{W}{g}\right)v_2^2$$

$$\Rightarrow \left(\frac{v_A^2}{2g}\right) + R - R\cos(\hat{\theta}) = \left(\frac{1}{2g}\right)g\,R\cos(\hat{\theta}) = \tfrac{1}{2}R\cos(\hat{\theta}) \Rightarrow \tfrac{3}{2}R\cos(\hat{\theta}) = \left(\frac{v_A^2}{2g}\right) + R$$

$$\Rightarrow \cos(\hat{\theta}) = \frac{2}{3R}\left(\frac{v_A^2}{2g} + R\right) = 0.749482 \Rightarrow \hat{\theta} \approx 41.5 \text{ (deg)}$$