

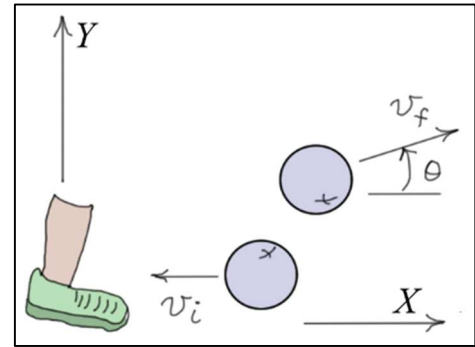
Elementary Dynamics Example #24: (Impulse & Momentum)

Given: $v_i = 5$ (ft/s), $v_f = 10$ (ft/s), $\theta = 30$ (deg)

$\Delta t = 0.01$ (sec), $W_B = 0.5$ (lb)

Find: a) average force exerted by the player on the ball
b) maximum force exerted by the player on the ball assuming a triangular force profile

Solution: (using the **principle of impulse & momentum**)



$$a) \quad \boxed{\vec{L}_1 + \vec{L}_{1 \rightarrow 2} = \vec{L}_2} \quad \text{with} \quad \boxed{\vec{L}_1 = -\left(\frac{W_B}{g}\right)v_i \vec{i}} \quad \text{and} \quad \boxed{\vec{L}_2 = \left(\frac{W_B}{g}\right)v_f (\cos(\theta) \vec{i} + \sin(\theta) \vec{j})}$$

$$X \text{ direction:} \quad \boxed{-\left(\frac{W_B}{g}\right)v_i + (F_x)_{\text{avg}} \Delta t = \left(\frac{W_B}{g}\right)v_f \cos(\theta)}$$

$$\Rightarrow \quad \boxed{(F_x)_{\text{avg}} = \left(\frac{W_B}{g \Delta t}\right)(v_i + v_f \cos(\theta)) \approx 21.2116 \approx 21.2 \text{ (lb)}}$$

$$Y \text{ direction:} \quad \boxed{0 + (F_y)_{\text{avg}} \Delta t = \left(\frac{W_B}{g}\right)v_f \sin(\theta)}$$

$$\Rightarrow \quad \boxed{(F_y)_{\text{avg}} = \left(\frac{W_B}{g \Delta t}\right)v_f \sin(\theta) \approx 7.76398 \approx 7.76 \text{ (lb)}}$$

$$\left. \begin{array}{l} (F_x)_{\text{avg}} \approx 21.2 \text{ (lb)} \\ (F_y)_{\text{avg}} \approx 7.76 \text{ (lb)} \end{array} \right\} \quad \boxed{|F_{\text{avg}}| \approx 22.6 \text{ (lb)}}$$

Note: In the above calculation, we **neglected** the impulse of the **weight force** of the ball. If we include the weight force in our calculation, the equation in the Y direction becomes:

$$Y \text{ direction:} \quad \boxed{\left((F_y)_{\text{avg}} - W_B\right) \Delta t = \left(\frac{W_B}{g}\right)v_f \sin(\theta)}$$

$$\Rightarrow \quad \boxed{(F_y)_{\text{avg}} = \left(\frac{W_B}{g \Delta t}\right)v_f \sin(\theta) + W_B \approx 8.26 \text{ (lb)}}$$

$$\left. \begin{array}{l} (F_x)_{\text{avg}} \approx 21.2 \text{ (lb)} \\ (F_y)_{\text{avg}} \approx 8.26 \text{ (lb)} \end{array} \right\} \quad \boxed{|F_{\text{avg}}| \approx 22.8 \text{ (lb)}}$$

b) If the force profile is assumed to be **triangular**, the **maximum** force can be calculated. In this case, the impulse of the force is

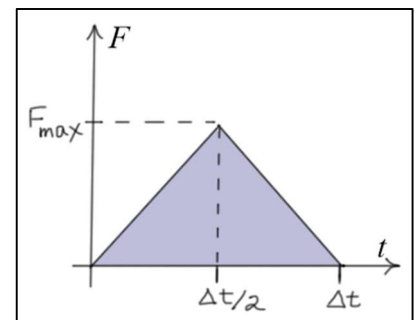
$$\boxed{\int_0^{\Delta t} F dt = \frac{1}{2}(\Delta t) F_{\text{max}}}$$

Ignoring the effect of the weight force gives

$$\boxed{(F_x)_{\text{max}} = \left(\frac{2W_B}{g \Delta t}\right)(v_i + v_f \cos(\theta)) \approx 42.4 \text{ (lb)}}$$

$$\boxed{(F_y)_{\text{max}} = \left(\frac{2W_B}{g \Delta t}\right)v_f \sin(\theta) \approx 15.5 \text{ (lb)}}$$

$$\left. \begin{array}{l} (F_x)_{\text{max}} \approx 42.4 \text{ (lb)} \\ (F_y)_{\text{max}} \approx 15.5 \text{ (lb)} \end{array} \right\} \quad \boxed{|F_{\text{max}}| \approx 45.2 \text{ (lb)}}$$



Using this approach, the **maximum force** is found to be **twice** the **average force**.