

## Introductory Control Systems

### First-Order System Step Response

- The general form of a transfer function for a first-order system is  $G(s) = \frac{K}{s + a}$ .
- The **time constant** associated with this system is  $\tau = 1/a$ . The time constant tells how quickly the system responds. For example, if  $a = 1$ , then the system responds on the order of  $\tau = 1$  second; however, if  $a = 100$ , then the system responds on the order of  $\tau = 0.01$  seconds. So, systems that **respond quickly** have **large** values of  $a$ , and systems that **respond slowly** have **small** values of  $a$ .
- The **2% settling time** for a first order system is approximately  $T_s = 4\tau = 4/a$ . This represents the amount of time required for the system to reach and stay within 2% of the final value.
- The plot below shows the **step response** for three different systems with  $a$  values of 0.5, 1, and 5. Setting  $K = a$  allows each system to attain a final value of 1, but note that the larger the value of  $a$ , the more quickly the system reaches the 98% value.
- Using the formula above, the 2% settling times for the three systems are estimated to be 4/5, 4, and 8 seconds, respectively. In the MATLAB plot below, the red, green, and blue dots represent MATLAB's estimates of these times.

