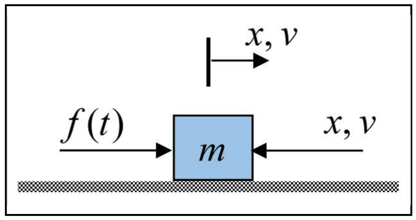
**Introductory Control Systems**
**Proportional Control of a First Order System**

**Example System: Simple Speed Control System**

Consider a ***car*** of ***mass*** $m$ traveling along a road with ***wind resistance*** (proportional to the speed of the car) as shown in the diagram. Applying Newton's 2nd law in the direction of travel and neglecting friction, write



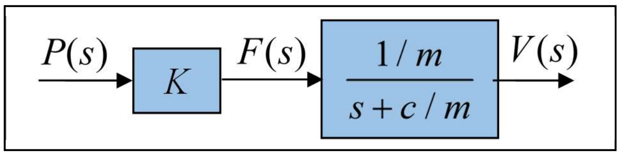

$$\overset{+}{\rightarrow}\sum F = f(t) - cv = m\dot{v} \qquad \Rightarrow \boxed{m\dot{v} + cv = f(t)}$$

Using ***Laplace transforms***, the ***transfer function*** for the system is

$$\boxed{\frac{V}{F}(s) = \frac{1/m}{s + c/m}} \tag{1}$$

**Open-Loop, Proportional Speed Control**

Now consider ***proportional, open-loop speed control*** of the car as indicated in the block diagram. The system ***transfer function*** is




$$\boxed{\frac{V}{P}(s) = \frac{K/m}{s + c/m}} \tag{2}$$

The ***final value*** due to a ***unit step*** input $P(s) = 1/s$ is $\boxed{v_{ss} = (K/m)/(c/m) = K/c}$.

Fig. 1 shows the step response of this system for $K = 300$, 600, and 900 using the parameters shown in Eq. (3) below. Note the value of $K$ ***affects*** the ***magnitude*** of the response, but it ***does not affect*** how long the car takes to reach a new final speed. That is, it ***does not affect*** the system's ***settling time***.

$$\boxed{\begin{aligned} m &= 100 \text{ slugs} \\ c &= 20 \text{ (lb-s/ft)} \end{aligned}} \tag{3}$$

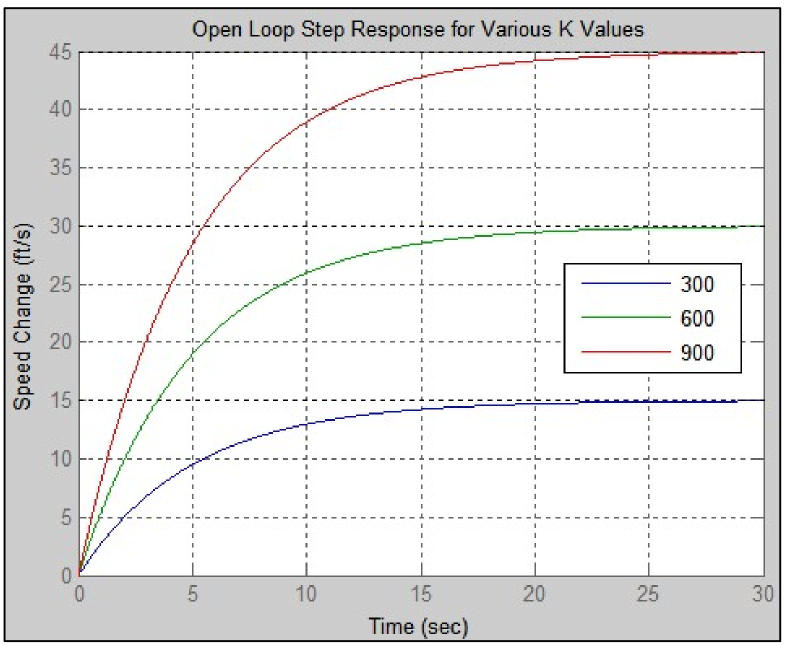
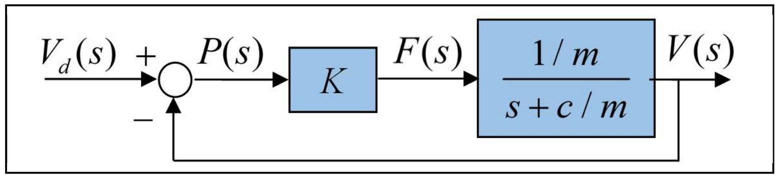



Figure 1. Open-Loop Step Response for Various $K$ Values

## Closed-Loop, Proportional Speed Control

Finally, consider the ***proportional, closed-loop speed control*** of the car as indicated in the block diagram. The transfer function of this system is




$$\frac{V}{V_d}(s) = \frac{K/m}{s + c/m + K/m} = \frac{K/m}{s + (c+K)/m} \quad (4)$$

The system input is $V_d$ the desired speed, and the output is the actual speed. The final value due to a *step input* $V_d(s) = 1/s$ is $\boxed{v_{ss} = (K/m)/\left((c+K)/m\right) = K/(c+K)}$. Fig. 2 shows the step response of the system for $K = 300$, 600, and 900. Note that the value of $K$ ***affects both*** the ***magnitude*** of the response and the ***time*** it takes the car to reach a new final speed. ***Higher*** values of $K$ give ***shorter*** settling times.

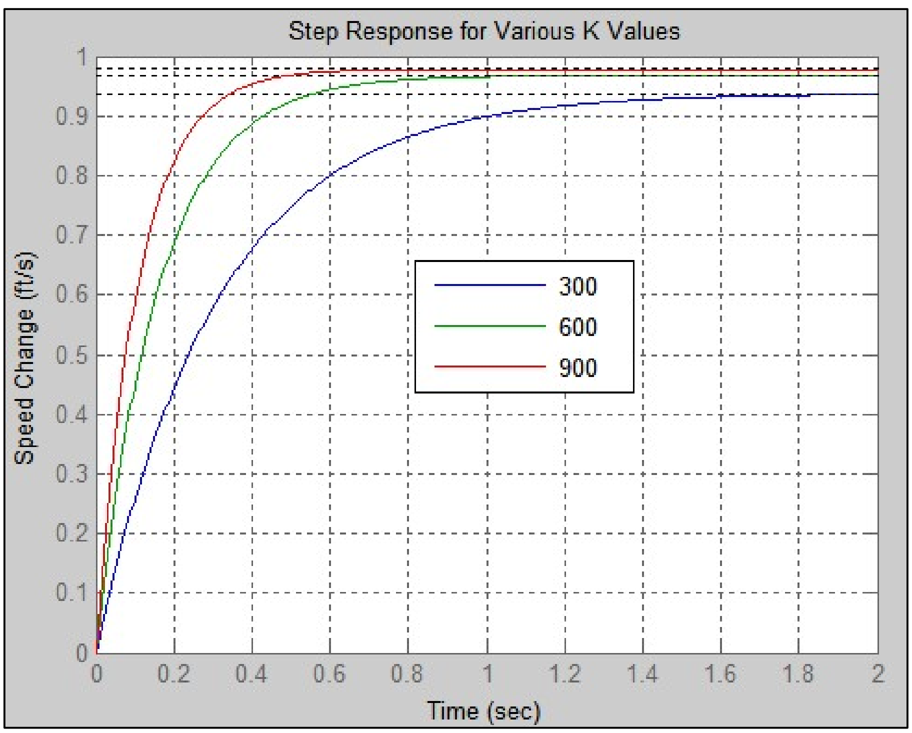



Figure 2. Closed-Loop Step Response for Various $K$ Values

In theory, the value of $K$ could be ***increased*** further to make the steady state response $(v_{ss})$ closer to the commanded value $(=1\ \text{(ft/s)})$ and the settling time smaller and smaller. However, as these changes are made, the ***force*** required to move the car becomes higher and higher. Fig. 3 shows the driving force $f(t)$ associated with the ***unit step responses*** shown in Fig. 2. Clearly, higher velocity commands and higher gains will cause the ***forces*** to eventually become ***unrealistic***.

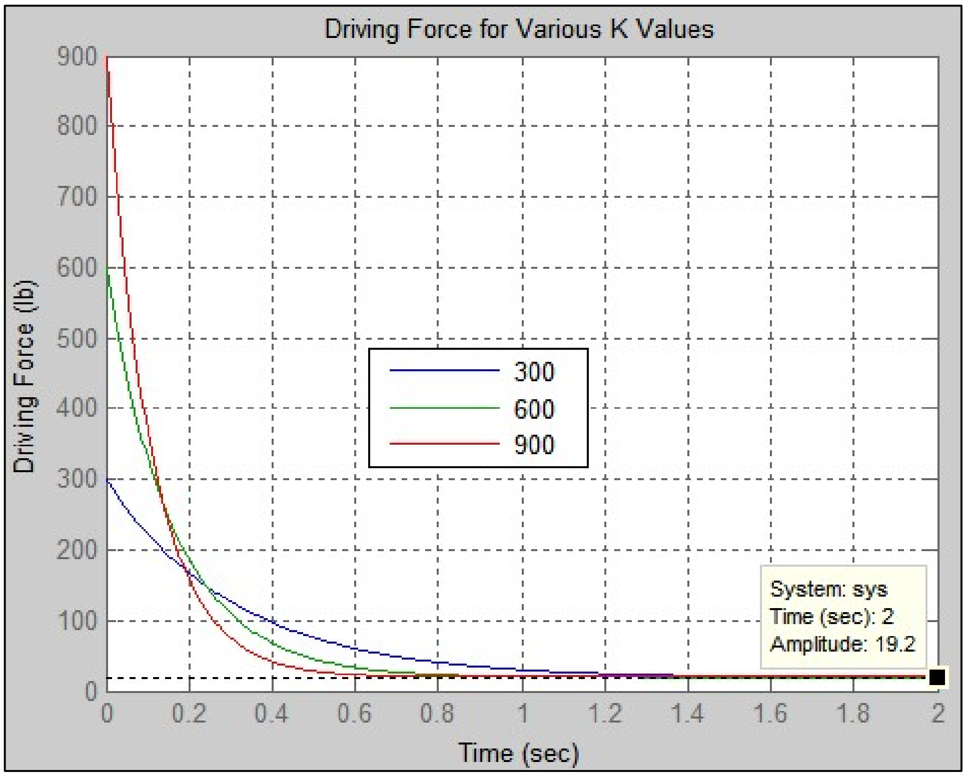



Figure 3. Driving Force for Closed-Loop Step Response for Various Gains