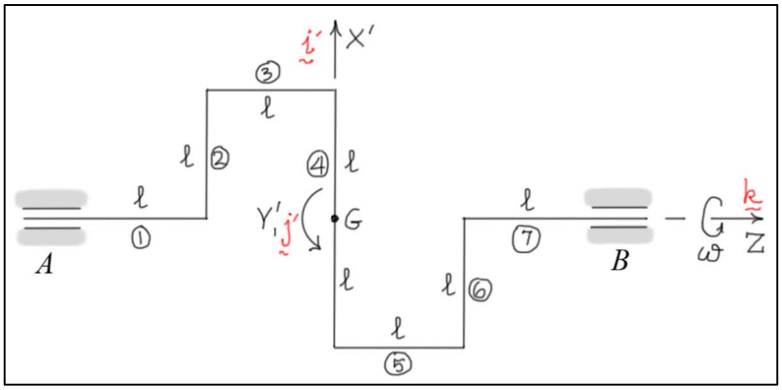
**Intermediate Dynamics**
**Bearing Loads on a Simple Crank Shaft**

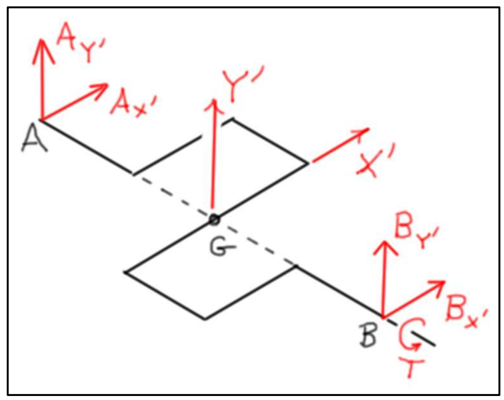
The figure shows a ***simple crank shaft*** consisting of ***seven*** segments, each considered to be a ***slender bar***. Each segment of length $\ell$ has mass $m$. There are six segments of length $\ell$ and one segment of length $2\ell$ (segment 4).




The ***mass center*** of the system is $G$ and is located on the axis of rotation. In previous notes, $\underset{\sim}{H}_G$ the ***angular momentum*** of the system was found to be

$$\boxed{\underset{\sim}{H}_G = 2m\ell^2\omega\,\underset{\sim}{i}' + (\tfrac{10}{3})m\ell^2\omega\,\underset{\sim}{k}}$$

The ***bearing loads*** at $A$ and $B$ can be found by applying the ***Newton/Euler equations of motion*** to the ***free-body*** diagram shown at the right.




$$\begin{aligned}
&\sum \underset{\sim}{F} = m\,{}^R\underset{\sim}{a}_G = \underset{\sim}{0}\\
&\sum \underset{\sim}{M}_A = \left(\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G\right) + \left(\underset{\sim}{r}_{G/A} \times m\,{}^R\underset{\sim}{a}_G\right)\\
&\qquad\quad = \left(\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B\right) + \left({}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G\right)
\end{aligned}$$

The terms on the left and right sides of the moment equation can be calculated as follows.

$$\sum \underset{\sim}{M}_A = \left(\underset{\sim}{r}_{B/A} \times \underset{\sim}{F}_B\right) + T\,\underset{\sim}{k} = -\left(4\ell B_{Y'}\right)\underset{\sim}{i}' + \left(4\ell B_{X'}\right)\underset{\sim}{j}' + T\,\underset{\sim}{k}$$

$$\underset{\approx}{I}_G \cdot {}^R\underset{\sim}{\alpha}_B = 2m\ell^2\dot{\omega}\,\underset{\sim}{i}' + (\tfrac{10}{3})m\ell^2\dot{\omega}\,\underset{\sim}{k}$$

$${}^R\underset{\sim}{\omega}_B \times \underset{\sim}{H}_G = \omega\,\underset{\sim}{k} \times \underset{\sim}{H}_G = 2m\ell^2\omega^2\,\underset{\sim}{j}'$$

Substituting these results into the equations above gives the following equations of motion.

<u>Force Equations</u>:

$$\boxed{\begin{aligned} A_{X'} + B_{X'} &= 0\\ A_{Y'} + B_{Y'} &= 0\end{aligned}}$$

Moment Equations:

$$\boxed{\begin{aligned} -4\ell B_{Y'} &= 2m\ell^2\dot{\omega} \\ 4\ell B_{X'} &= 2m\ell^2\omega^2 \\ T &= \tfrac{10}{3}m\ell^2\dot{\omega} \end{aligned}}$$

Solving these equations gives the following results.

$$\boxed{\begin{aligned} B_{X'} &= \tfrac{1}{2}m\ell\omega^2 = -A_{X'} \\ B_{Y'} &= -\tfrac{1}{2}m\ell\dot{\omega} = -A_{Y'} \\ T &= \tfrac{10}{3}m\ell^2\dot{\omega} \end{aligned}}$$

Notes:

1. Even if the system is rotating at a ***constant rate*** (so that $\dot{\omega} = 0$), the ***in-plane bearing loads*** ($A_{X'}$ and $B_{X'}$) are still ***non-zero*** due to the ***in-plane asymmetry*** of the crank shaft.
2. The magnitudes of these forces ***increase*** with the ***square*** of the ***rotational speed***.