

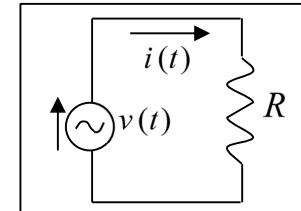
Elementary Engineering Mathematics

Application of Integration in Electrical Engineering

Current–Voltage Relationships for Resistors, Capacitors, and Inductors

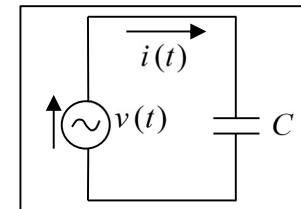
The voltage across and the current through a *resistor* are related by its resistance.

$$v(t) = R i(t) \quad \text{or} \quad i(t) = v(t)/R$$



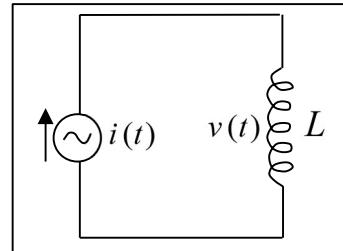
The voltage across and the current through a *capacitor* of capacitance C are related by the following equations.

$$i(t) = C \frac{dv}{dt} \quad \text{or} \quad v(t) = \frac{1}{C} \int i(t) dt$$



The voltage across and the current through an *inductor* of inductance L are related by the following equations.

$$v(t) = L \frac{di}{dt} \quad \text{or} \quad i(t) = \frac{1}{L} \int v(t) dt$$



Example 1:

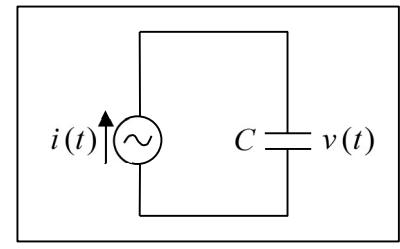
Given: A current $i(t) = 5(1 - e^{-3t})$ (amps) is applied to a capacitor with $C = 0.5$ (f).

Find: $v(t)$ the voltage across the capacitor. Assume $v(0) = 0$.

Solution:

The voltage can be found by integrating the current.

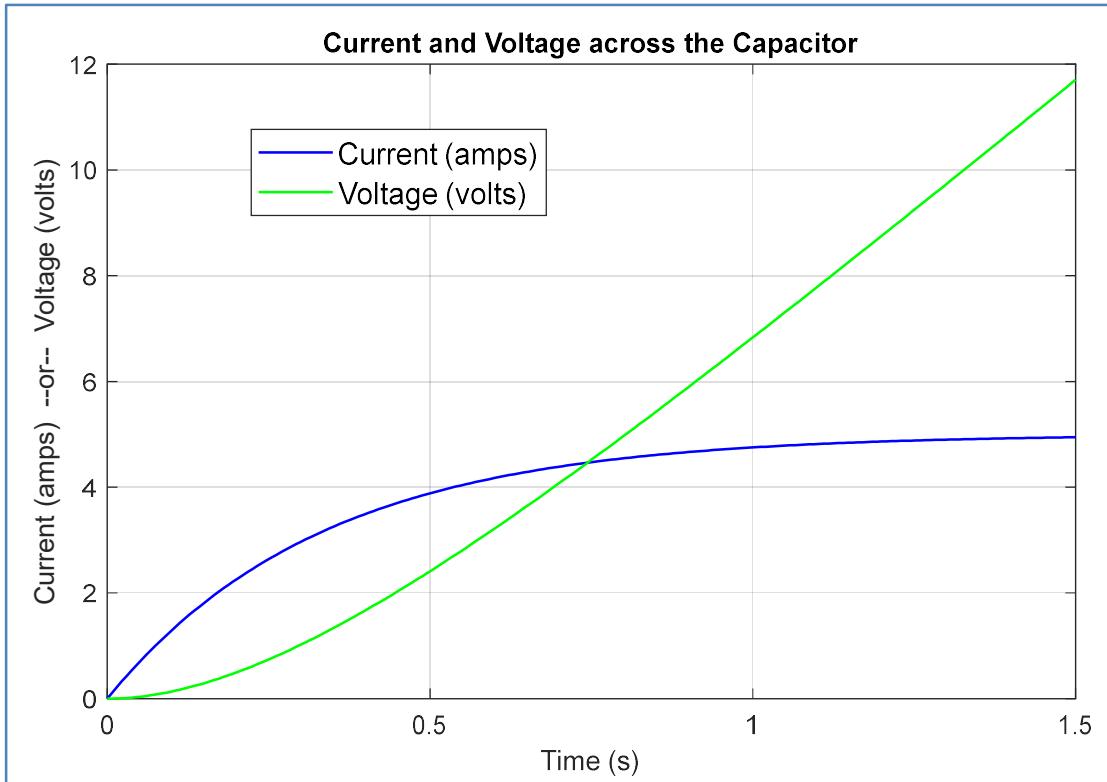
$$v(t) = \frac{1}{C} \int i(t) dt = 2 \int 5(1 - e^{-3t}) dt = 10 \left(t + \frac{1}{3} e^{-3t} \right) + D$$



To find the constant D , apply the *initial condition*

$$v(0) = 0 = 10 \left(\frac{1}{3} e^0 \right) + D \quad \Rightarrow \quad D = -\frac{10}{3} \quad \Rightarrow \quad v(t) = 10 \left(-\frac{1}{3} + t + \frac{1}{3} e^{-3t} \right) \text{ (volts)}$$

$$\text{Check: } i(t) = C \frac{dv}{dt} = 0.5 \frac{d}{dt} \left[10 \left(-\frac{1}{3} + t + \frac{1}{3} e^{-3t} \right) \right] = 5 \left(0 + 1 - e^{-3t} \right) = 5 \left(1 - e^{-3t} \right) \quad \checkmark$$



Example 2:

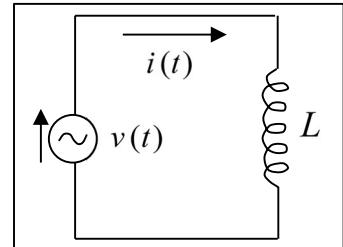
Given: A voltage $v(t) = 120 \sin(60\pi t)$ (volts) is applied to an inductor with $L = 500$ (mh).

Find: Current $i(t)$ through the inductor. Assume $i(0) = 0$.

Solution:

To find the current, integrate the voltage.

$$i(t) = \frac{1}{L} \int v(t) dt = 2 \int 120 \sin(60\pi t) dt = 240 \left[\frac{-\cos(60\pi t)}{60\pi} \right] + D$$



To find the constant D , apply the **initial condition** $i(0) = 0$.

$$i(0) = 0 = D - \frac{4}{\pi} = 0 \Rightarrow D = \frac{4}{\pi} \Rightarrow i(t) = \frac{4}{\pi} [1 - \cos(60\pi t)] \text{ (amps)}$$

$$\text{Check: } v(t) = L \frac{di}{dt} = 0.5 \frac{d}{dt} \left(\frac{4}{\pi} (1 - \cos(60\pi t)) \right) = \frac{2}{\pi} (60\pi \sin(60\pi t)) = 120 \sin(60\pi t)$$

✓

