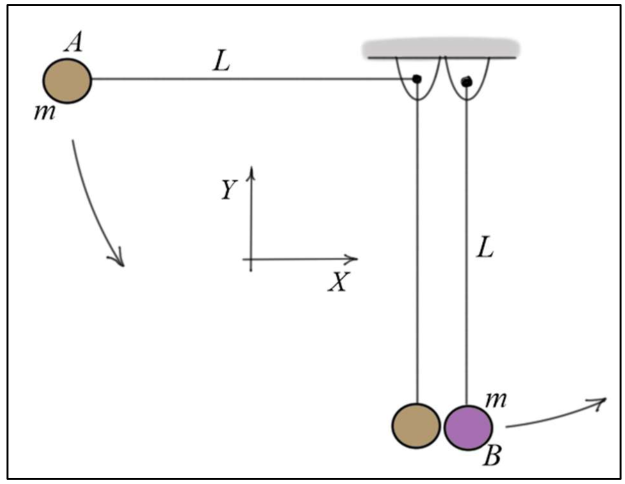
Elementary Dynamics Example #27: (Conservation of Momentum, Impact)

Given: $m_A = m_B = m$, $e = 0.6$,
m_A is released from rest at the top and strikes m_B at the bottom

Find: h the ***maximum height*** to which m_B swings



Solution: (***conservation of momentum and energy***)

1. Using ***conservation of energy***, the velocity of m_A as it strikes m_B is found.

$$\boxed{\cancel{K_1} + V_1 = K_2 + \cancel{V_2}} \quad \text{(datum at the bottom)}$$

So,

$$\boxed{V_1 = mgL = K_2 = \tfrac{1}{2}m\left(v_{A2}\right)^2} \quad \Rightarrow \boxed{v_{A2} = \sqrt{2gL}} \quad \text{(just before impact)}$$

2. Using conservation of momentum: (in X direction, 2 – just before impact, 3 – just after)

$$\boxed{\cancel{m}\left(v_{A2}\right) + \underbrace{m\left(v_{B2}\right)}_{\text{zero}} = \cancel{m}\left(v_{A3}\right) + \cancel{m}\left(v_{B3}\right)}$$

$$\Rightarrow \boxed{v_{A3} + v_{B3} = v_{A2} = \sqrt{2gL}}$$

Using the impact (restitution) equation:

$$\boxed{\frac{v_{A3} - v_{B3}}{\cancel{v_{B2}} - v_{A2}} = e} \quad \Rightarrow \boxed{v_{A3} - v_{B3} = -e\,v_{A2} = -0.6\sqrt{2gL}}$$

Solving the two equations simultaneously gives:

$$\boxed{v_{A3} = 0.2\sqrt{2gL}}, \quad \boxed{v_{B3} = 0.8\sqrt{2gL}}$$

3. Using conservation of energy for m_B after impact:

$$\boxed{K_3 + \cancel{V_3} = \cancel{K_4} + V_4} \quad \text{(datum at the bottom)}$$

So,

$$\boxed{K_3 = \tfrac{1}{2}m\left(v_{B3}\right)^2 = \cancel{\tfrac{1}{2}}m\left(0.8\right)^2\left(\cancel{2}gL\right) = 0.64mgL = V_4 = mgh} \quad \Rightarrow \boxed{h = 0.64L}$$

Question: What would the result be if $e = 1$?

Answer: Using the above approach, it is easy to show that, in this case, $h = L$. This makes sense if you recall that ***kinetic energy*** of the impact is ***conserved*** if $e = 1$.