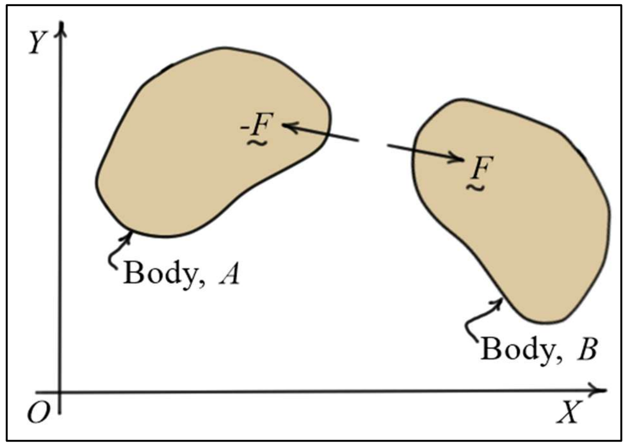
Elementary Dynamics
Conservation of Momentum and Impact for Rigid Bodies in Two Dimensions

Conservation of Linear Momentum

If the ***net impulse*** on a ***rigid body*** is ***zero*** over the time interval $t_1 \to t_2$, the ***linear momentum*** of the ***body*** is ***conserved*** (i.e. the mass center moves with constant velocity). If the ***net impulse*** on a ***system*** of ***rigid bodies*** is ***zero*** over the time interval $t_1 \to t_2$, the ***linear momentum*** of the ***system*** of bodies is ***conserved***.

For example, consider two ***colliding bodies***. If the ***impulsive*** forces $\underset{\sim}{F}$ and $-\underset{\sim}{F}$ act over a ***short*** time interval $t_1 \to t_2$, the principle of linear impulse and momentum can be applied to each body over that interval. Because the impulses of $\underset{\sim}{F}$ and $-\underset{\sim}{F}$ are ***equal*** and ***opposite***, the ***linear momentum*** of the ***system*** of the two bodies is ***conserved***. That is,



$$\boxed{\left(m_A \underset{\sim}{v}_A\right)_1 + \left(m_B \underset{\sim}{v}_B\right)_1 = \left(m_A \underset{\sim}{v}_A\right)_2 + \left(m_B \underset{\sim}{v}_B\right)_2}$$

Conservation of Angular Momentum

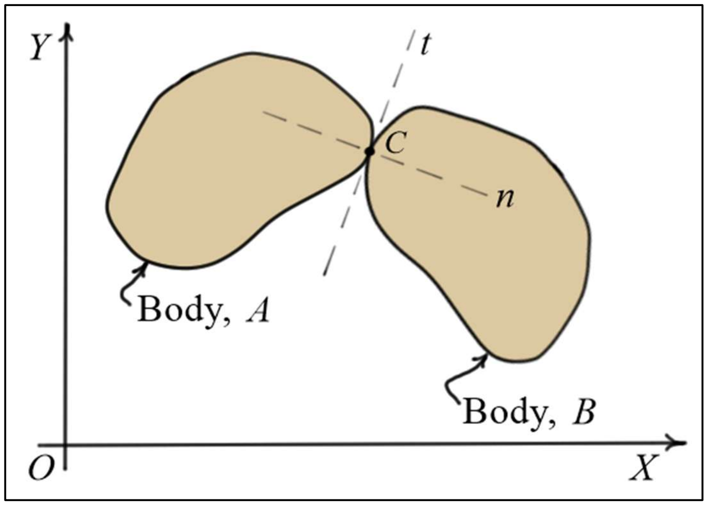
If the ***net angular impulse*** on a ***rigid body*** is ***zero*** over the time interval $t_1 \to t_2$, the ***angular momentum*** of the ***body*** is ***conserved*** (i.e. the body rotates with constant angular velocity). If the ***net angular impulse*** on a ***system*** of ***rigid bodies*** is ***zero*** over the time interval $t_1 \to t_2$, the ***angular momentum*** of the ***system*** of bodies is ***conserved***. For example, consider again two colliding bodies. The ***net angular impulse*** of the two ***impulsive*** forces $\underset{\sim}{F}$ and $-\underset{\sim}{F}$ about the ***fixed point*** O is zero, so the ***angular momentum*** of the ***system*** about O is ***conserved*** during the ***impact***. That is,

$$\boxed{\left(\underset{\sim}{H}_O\right)_{A1} + \left(\underset{\sim}{H}_O\right)_{B1} = \left(\underset{\sim}{H}_O\right)_{A2} + \left(\underset{\sim}{H}_O\right)_{B2}}$$

Recall the angular momentum of a body about a fixed-point O is calculated as follows.

$$\boxed{\underset{\sim}{H}_O = I_G \underset{\sim}{\omega} + (\underset{\sim}{r}_G \times m \underset{\sim}{v}_G)}$$

Coefficient of Restitution



Consider two colliding bodies A and B. At the ***contact point*** C, the directions n and t are ***normal*** and ***tangent*** to the colliding surfaces. If the ***friction forces*** resulting from the impact are ***negligible***, it can be shown that the ***relative velocities*** of the points of contact on the two bodies in the n-direction can be related through e the coefficient of restitution as follows:

$$\boxed{e = \frac{(v_{CB})_{n2} - (v_{CA})_{n2}}{(v_{CA})_{n1} - (v_{CB})_{n1}}}$$

Here, $(v_{CA})_{n1}$ and $(v_{CB})_{n1}$ represent the ***velocities*** of the ***contact points*** on bodies A and B in the n-direction ***before impact*** (time, t_1), and $(v_{CA})_{n2}$ and $(v_{CB})_{n2}$ represent the ***velocities*** of the ***contact points*** on bodies A and B in the n-direction ***after impact*** (time, t_2).