

Elementary Dynamics Example #28: (Conservation of Momentum, Impact)

Given: $W_A = 13.2 \times 10^{-3}$ (lb), $W_B = 6.6 \times 10^{-3}$ (lb), $e = 0.65$

- velocities shown are just prior to collision
- disks are sliding on a smooth horizontal surface

Find: velocities of A and B just after impact

Solution:

- velocities are conserved in the ξ_y direction
- linear momentum is conserved in the ξ_x direction

Before impact: (state 1)

$$v_A = 2 \xi = 2(\sin(30)\xi_x + \cos(30)\xi_y) \text{ (ft/s)}, \quad v_B = -3 \xi = -3(\sin(30)\xi_x + \cos(30)\xi_y) \text{ (ft/s)}$$

Collision:

ξ_y direction:

$$(v_{Ay})_2 = (v_{Ay})_1 = 2\cos(30) \approx 1.7321 \quad \text{and} \quad (v_{By})_2 = (v_{By})_1 = -3\cos(30) \approx -2.5981 \text{ (ft/s)}$$

ξ_x direction: (conservation of linear momentum and restitution equation)

$$m_A(v_{Ax})_2 + m_B(v_{Bx})_2 = m_A(v_{Ax})_1 + m_B(v_{Bx})_1 = m_A(2\sin(30)) + m_B(-3\sin(30))$$

$$\Rightarrow m_A(v_{Ax})_2 + m_B(v_{Bx})_2 = (2m_A - 3m_B)\sin(30)$$

Multiplying both sides by g gives:

$$W_A(v_{Ax})_2 + W_B(v_{Bx})_2 = (2W_A - 3W_B)\sin(30) = 1.0248 \times 10^{-4} g$$

$$\frac{(v_{Bx})_2 - (v_{Ax})_2}{(v_{Ax})_1 - (v_{Bx})_1} = e \Rightarrow \frac{-(v_{Ax})_2 + (v_{Bx})_2}{(v_{Ax})_1 - (v_{Bx})_1} = e[(v_{Ax})_1 - (v_{Bx})_1]$$

$$= 0.65[2\sin(30) + 3\sin(30)] \approx 1.625$$

Simultaneous equations:

$$\begin{cases} 13.2(v_{Ax})_2 + 6.6(v_{Bx})_2 = 0.10248g \\ -(v_{Ax})_2 + (v_{Bx})_2 = 1.625 \end{cases} \Rightarrow \begin{cases} (v_{Ax})_2 = -0.375 \text{ (ft/s)} \\ (v_{Bx})_2 = 1.25 \text{ (ft/s)} \end{cases}$$

Notes:

- Incoming angle of each of the masses with the ξ_y direction is equal to 30 degrees.
- Outgoing angles with the ξ_y direction:

$$A: \theta_A = \tan^{-1}\left(\frac{0.375}{1.7321}\right) \approx 12.2 \text{ (deg) CCW} \quad B: \theta_B = \tan^{-1}\left(\frac{1.25}{2.5981}\right) \approx 25.7 \text{ (deg) CCW}$$

The smaller mass exits the collision at close to 30 degrees off the ξ_y direction, but the larger mass is well below 30 degrees.