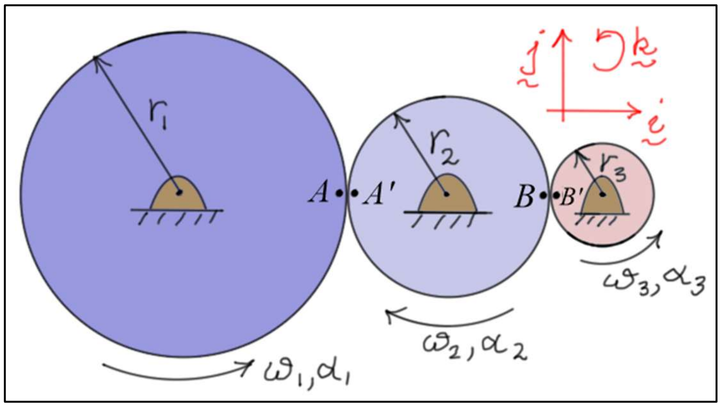
Elementary Dynamics Example #29: (Rigid Body Kinematics – Pure Rotational Motion)

Given: $r_1 = 6$ (in), $r_2 = 4$ (in), $r_3 = 2$ (in)

$\omega_1 = 1000$ (rpm) CCW

$\alpha_1 = 5$ (r/s^2) CCW

- no slipping between the gears
- radii represent the pitch circles



Find: a) ω_2, ω_3

b) α_2, α_3

c) $\underset{\sim}{a}_A$, $\underset{\sim}{a}_{A'}$

Solution:

a) Because the gears do not slip relative to each other, $\underset{\sim}{v}_A = \underset{\sim}{v}_{A'}$ and $\underset{\sim}{v}_B = \underset{\sim}{v}_{B'}$.

$$\underset{\sim}{v}_A = r_1\omega_1\,\underset{\sim}{j} = \underset{\sim}{v}_{A'} = r_2\omega_2\,\underset{\sim}{j} \;\Rightarrow\; \omega_2 = \left(\frac{r_1}{r_2}\right)\omega_1 = \tfrac{3}{2}(1000) = 1500 \text{ (rpm) CW}$$

$$\underset{\sim}{v}_B = -r_2\omega_2\,\underset{\sim}{j} = \underset{\sim}{v}_{B'} = -r_3\omega_3\,\underset{\sim}{j} \;\Rightarrow\; \omega_3 = \left(\frac{r_2}{r_3}\right)\omega_2 = \tfrac{4}{2}(1500) = 3000 \text{ (rpm) CCW}$$

Each of these rates can be expressed in radians/seconds by multiplying by $\frac{2\pi}{60}$. For example,

$$1000\frac{\cancel{\text{rev}}}{\cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{ sec}} \times \frac{2\pi\text{ rad}}{1\ \cancel{\text{rev}}} = 1000\left(\tfrac{2\pi}{60}\right) \approx 104.7 \text{ (rad/s)}$$

b) Because the gears do not slip, the accelerations of the contact points ***tangent*** to the contacting surfaces must be equal. That is, $\left(\underset{\sim}{a}_A\right)_{\tan} = \left(\underset{\sim}{a}_{A'}\right)_{\tan}$ and $\left(\underset{\sim}{a}_B\right)_{\tan} = \left(\underset{\sim}{a}_{B'}\right)_{\tan}$.

$$\left(\underset{\sim}{a}_A\right)_{\tan} = r_1\alpha_1\,\underset{\sim}{j} = \left(\underset{\sim}{a}_{A'}\right)_{\tan} = r_2\alpha_2\,\underset{\sim}{j} \;\Rightarrow\; \alpha_2 = \left(\frac{r_1}{r_2}\right)\alpha_1 = \tfrac{6}{4}(5) = 7.5 \text{ (r/s}^2\text{) CW}$$

$$\left(\underset{\sim}{a}_B\right)_{\tan} = -r_2\alpha_2\,\underset{\sim}{j} = \left(\underset{\sim}{a}_{B'}\right)_{\tan} = -r_3\alpha_3\,\underset{\sim}{j} \;\Rightarrow\; \alpha_3 = \left(\frac{r_2}{r_3}\right)\alpha_2 = \tfrac{4}{2}(7.5) = 15 \text{ (r/s}^2\text{) CCW}$$

c) For A, $\underset{\sim}{e}_r = \underset{\sim}{i}$ and $\underset{\sim}{e}_\theta = \underset{\sim}{j}$. For A', $\underset{\sim}{e}_r = -\underset{\sim}{i}$ and $\underset{\sim}{e}_\theta = \underset{\sim}{j}$. Using the equations for radial and transverse acceleration for constant radius motion,

$$\underset{\sim}{a}_A = -r_1\omega_1^2\,\underset{\sim}{i} + r_1\alpha_1\,\underset{\sim}{j} = \left(-\left(\tfrac{6}{12}\right)(104.72)^2\right)\underset{\sim}{i} + \left(\left(\tfrac{6}{12}\right)5\right)\underset{\sim}{j} \approx -5480\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j} \text{ (ft/s}^2\text{)}$$

$$\underset{\sim}{a}_{A'} = r_2\omega_2^2\,\underset{\sim}{i} + r_2\alpha_2\,\underset{\sim}{j} = \left(-\left(\tfrac{4}{12}\right)\left(\tfrac{1500(2\pi)}{60}\right)^2\right)\underset{\sim}{i} + \left(\left(\tfrac{4}{12}\right)7.5\right)\underset{\sim}{j} \approx 8225\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j} \text{ (ft/s}^2\text{)}$$

Note that the accelerations of A and A' are ***equal*** only in the ***tangential*** direction, $\underset{\sim}{j}$. Their accelerations in the $\underset{\sim}{i}$ direction ***differ*** in both ***magnitude*** and ***sign***.