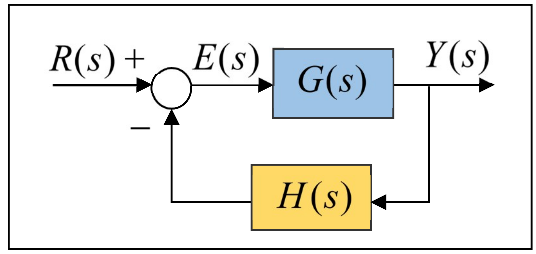
Introductory Control Systems
System Type and Steady-State Error

System Type

Consider the single loop feedback system with input $R(s)$ and output $Y(s)$ as shown at the right. The ***system type*** is determined by the form of the ***loop*** (or open loop) ***transfer function*** $GH(s)$. In general, the form of $GH(s)$ can be written as



Simple Closed Loop System

$$\boxed{GH(s) = \frac{N(s)}{s^n D(s)}} \tag{1}$$

where $N(s)$ and $D(s)$ represent polynomials in s, and s^n represents all powers of s that can be factored from the denominator. The number n determines the system type. It is said that the system is a "***type n***" system. As is shown below, the value of n determines the "type" of ***steady-state error*** the system will have for various input signals.

System Error Transfer Function and Steady-State Error

Using ***block diagram reduction***, it can be shown that the ***error transfer function*** for the single loop system is

$$\boxed{\frac{E}{R}(s) = \frac{1}{1+GH(s)}} \tag{2}$$

Combining equations (1) and (2) gives the general form

$$\boxed{\frac{E}{R}(s) = \frac{s^n D(s)}{\left(s^n D(s)\right) + N(s)}} \tag{3}$$

The ***steady state error*** is found using the ***final value theorem***. Steady state errors for type "0" and type "1" systems for ***step***, ***ramp***, and ***parabolic*** inputs are calculated below. Given these results, the results for type "2" and higher systems are easily deduced.

Type "0" System

The steady-state error of a type "0" system to ***step***, ***ramp***, and ***parabolic*** inputs can be calculated as follows:

Step Input:

$$e_{ss} = \lim_{s\to 0}\left(s\,E(s)\right) = \lim_{s\to 0}\left(s\cdot\frac{E}{R}\cdot\frac{1}{s}\right) = \lim_{s\to 0}\left(\frac{D(s)}{D(s)+N(s)}\right) = \frac{D(0)}{D(0)+N(0)} = \text{ finite value}$$

Ramp Input:

$$e_{ss} = \lim_{s\to 0}\left(s\,E(s)\right) = \lim_{s\to 0}\left(s\cdot\frac{E}{R}\cdot\frac{1}{s^2}\right) = \lim_{s\to 0}\left(\frac{D(s)}{s\left(D(s)+N(s)\right)}\right) = \frac{D(0)}{0} = \text{ infinite value}$$

Parabolic Input:

$$e_{ss} = \lim_{s\to 0}\left(s\,E(s)\right) = \lim_{s\to 0}\left(s\cdot\frac{E}{R}\cdot\frac{1}{s^3}\right) = \lim_{s\to 0}\left(\frac{D(s)}{s^2\left(D(s)+N(s)\right)}\right) = \frac{D(0)}{0} = \text{ infinite value}$$

Type "1" System

The steady-state error of a type "1" system to ***step***, ***ramp***, and ***parabolic*** inputs can be calculated as follows:

Step Input:

$$e_{ss} = \lim_{s\to 0}\left(s\,E(s)\right) = \lim_{s\to 0}\left(s\cdot\frac{E}{R}\cdot\frac{1}{s}\right) = \lim_{s\to 0}\left(\frac{s\,D(s)}{\left(s\,D(s)\right)+N(s)}\right) = \frac{0}{N(0)} = \text{ zero}$$

Ramp Input:

$$e_{ss} = \lim_{s\to 0}\left(s\,E(s)\right) = \lim_{s\to 0}\left(s\cdot\frac{E}{R}\cdot\frac{1}{s^2}\right) = \lim_{s\to 0}\left(\frac{D(s)}{sD(s)+N(s)}\right) = \frac{D(0)}{N(0)} = \text{ finite value}$$

Parabolic Input:

$$e_{ss} = \lim_{s\to 0}\left(s\,E(s)\right) = \lim_{s\to 0}\left(s\cdot\frac{E}{R}\cdot\frac{1}{s^3}\right) = \lim_{s\to 0}\left(\frac{D(s)}{s\left(s\,D(s)+N(s)\right)}\right) = \frac{D(0)}{0} = \text{ infinite value}$$

Summary: (following the pattern of the above calculations)

Type	Steady State System Error		
	Step	Ramp	Parabolic
0	finite	∞	∞
1	0	finite	∞
2	0	0	finite
3	0	0	0