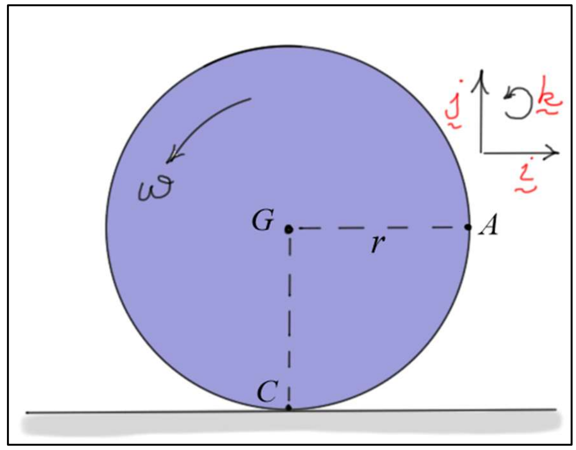
Elementary Dynamics Example #31: (Rigid Body Kinematics – Relative Velocity)



Given: r, ω

wheel rolls without slipping $\left(\underset{\sim}{v}_C = \underset{\sim}{0}\right)$.

Find: a) $\underset{\sim}{v}_G$ the velocity of G

b) $\underset{\sim}{v}_A$ the velocity of A

Solution:

a) Using the relative velocity equation for two points fixed on a rigid body,

$$\underset{\sim}{v}_G = \underset{\sim}{v}_C + \underset{\sim}{v}_{G/C} = \underbrace{\underset{\sim}{v}_C}_{\text{zero}} + \underset{\sim}{v}_{G/C} = \left[\underset{\sim}{\omega} \times \underset{\sim}{r}_{G/C}\right] = \omega \underset{\sim}{k} \times r\, \underset{\sim}{j}$$

$$\Rightarrow \boxed{\underset{\sim}{v}_G = -r\omega\, \underset{\sim}{i}}$$

b) Again, using the relative velocity equation,

$$\underset{\sim}{v}_A = \underset{\sim}{v}_C + \underset{\sim}{v}_{A/C} = \underbrace{\underset{\sim}{v}_C}_{\text{zero}} + \underset{\sim}{v}_{A/C} = \left[\underset{\sim}{\omega} \times \underset{\sim}{r}_{A/C}\right] = \omega \underset{\sim}{k} \times \left(r\, \underset{\sim}{i} + r\, \underset{\sim}{j}\right)$$

$$\Rightarrow \boxed{\underset{\sim}{v}_A = -r\omega\, \underset{\sim}{i} + r\omega\, \underset{\sim}{j}}$$

or,

$$\underset{\sim}{v}_A = \underset{\sim}{v}_G + \underset{\sim}{v}_{A/G} = -r\omega\, \underset{\sim}{i} + \underset{\sim}{v}_{A/G} = -r\omega\, \underset{\sim}{i} + \left[\underset{\sim}{\omega} \times \underset{\sim}{r}_{A/G}\right] = -r\omega\, \underset{\sim}{i} + \left[\omega \underset{\sim}{k} \times r\, \underset{\sim}{i}\right]$$

$$\Rightarrow \boxed{\underset{\sim}{v}_A = -r\omega\, \underset{\sim}{i} + r\omega\, \underset{\sim}{j}} \quad \ldots \quad \text{same result}$$

Notes:

1. Because the wheel ***rolls without slipping***, the contact point has the ***same velocity*** as the contacting surface which, in this case, is ***zero***.
2. In the above form, the relative velocity equation can be used to relate the velocities of ***any two points*** on a rigid body in terms of its ***angular velocity***.
3. ***Velocity*** is a ***property*** that applies to the ***points*** of a body, whereas ***angular velocity*** is a ***property*** that applies to an ***entire body***, not the individual points of that body.