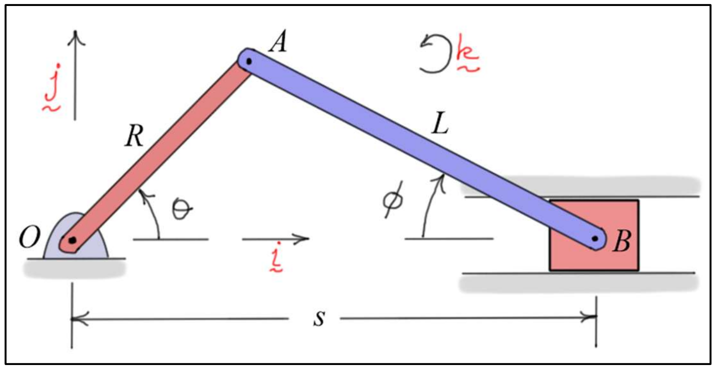
Elementary Dynamics Example #32: (Rigid Body Kinematics – Relative Velocity)

Given: $R = 3$ (in), $L = 6$ (in), $\theta = 30$ (deg)

$\omega_{OA} = \dot{\theta} = 100$ (rpm) (CCW)

Find: ω_{AB}, $v_B = \dot{s}$



Solution:

From the triangle formed by the mechanism,

$$\boxed{R\sin(\theta) = L\sin(\phi)} \Rightarrow \boxed{\phi = \sin^{-1}\left(\frac{R\sin(\theta)}{L}\right)_{\theta=30\ (\text{deg})} = 14.4775\ (\text{deg})}$$

Using the relative velocity equations, write

$$\boxed{\underset{\sim}{v}_B = \underset{\sim}{v}_A + \underset{\sim}{v}_{B/A}}\ *$$

Here,

$$\boxed{\underset{\sim}{v}_B = v_B\,\underset{\sim}{i}}$$

$$\underset{\sim}{v}_A = \underbrace{\underset{\sim}{v}_O}_{\text{zero}} + \underset{\sim}{v}_{A/O} = \omega_{OA}\underset{\sim}{k} \times R\left(\cos(\theta)\,\underset{\sim}{i} + \sin(\theta)\,\underset{\sim}{j}\right) = R\omega_{OA}\left(-\sin(\theta)\,\underset{\sim}{i} + \cos(\theta)\,\underset{\sim}{j}\right)$$

$$= 3(100)\left(\tfrac{2\pi}{60}\right)\left(-\sin(30)\,\underset{\sim}{i} + \cos(30)\,\underset{\sim}{j}\right)$$

$$\Rightarrow \boxed{\underset{\sim}{v}_A = -15.708\,\underset{\sim}{i} + 27.207\,\underset{\sim}{j}\ (\text{in/s})}$$

$$\boxed{\underset{\sim}{v}_{B/A} = \omega_{AB}\underset{\sim}{k} \times L\left(\cos(\phi)\,\underset{\sim}{i} - \sin(\phi)\,\underset{\sim}{j}\right) = 6\omega_{AB}\left(\sin(\phi)\,\underset{\sim}{i} + \cos(\phi)\,\underset{\sim}{j}\right)}$$

Substituting into the relative velocity equation (*) give the following scalar equations:

$$\boxed{\begin{aligned} v_B &= -15.708 + 6\omega_{AB}\sin(\phi) \\ 0 &= \ \ 27.207 + 6\omega_{AB}\cos(\phi) \end{aligned}}$$

Solving for ω_{AB} and v_B gives

$$\omega_{AB} = -27.207 / \left(6\cos(\phi)\right) = -4.68321 \quad \Rightarrow \boxed{\underset{\sim}{\omega}_{AB} \approx -4.68\,\underset{\sim}{k}\ (\text{rad/s})}$$

$$v_B = -15.708 + 6\omega_{AB}\sin(\phi) = -22.7328 \quad \Rightarrow \boxed{\underset{\sim}{v}_B \approx -22.7\,\underset{\sim}{i}\ (\text{in/s}) \approx -1.89\,\underset{\sim}{i}\ (\text{ft/s})}$$

So, in the current position, link *AB* is ***rotating clockwise*** at 4.68 (rad/s) and *B* is moving to the ***left*** at 1.89 (ft/s).

Note: The signs of the variables ω_{AB} and v_B are found by solving the simultaneous scalar equations. They need not be known prior to solving the equations.