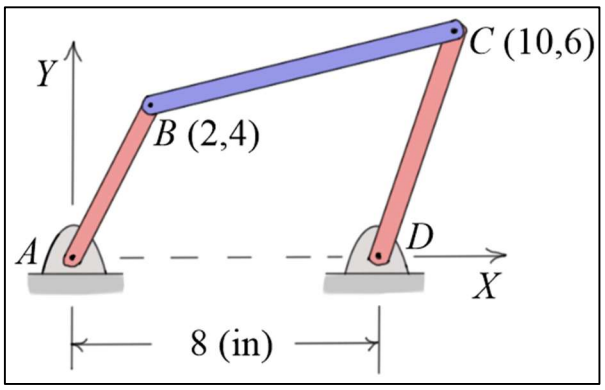
Elementary Dynamics Example #32a: (Rigid Body Kinematics – Relative Velocity)

Given: - coordinates are in inches

$\omega_{AB} = 10$ (rad/s) (CCW)




Find: $\underset{\sim}{\omega}_{BC}$, $\underset{\sim}{\omega}_{CD}$

Solution:

Using the relative velocity equation,

$$\boxed{\underset{\sim}{v}_C = \underset{\sim}{v}_B + \underset{\sim}{v}_{C/B}} \; *$$

Here,

$$\underset{\sim}{v}_C = \underset{\sim}{v}_{C/D} = \omega_{CD}\,\underset{\sim}{k} \times \underset{\sim}{r}_{C/D} = \omega_{CD}\,\underset{\sim}{k} \times \left(2\,\underset{\sim}{i} + 6\,\underset{\sim}{j}\right) = \omega_{CD}\left(-6\,\underset{\sim}{i} + 2\,\underset{\sim}{j}\right)$$

$$\underset{\sim}{v}_B = \underset{\sim}{v}_{B/A} = 10\,\underset{\sim}{k} \times \underset{\sim}{r}_{B/A} = 10\,\underset{\sim}{k} \times \left(2\,\underset{\sim}{i} + 4\,\underset{\sim}{j}\right) = -40\,\underset{\sim}{i} + 20\,\underset{\sim}{j}$$

$$\underset{\sim}{v}_{C/B} = \omega_{BC}\,\underset{\sim}{k} \times \underset{\sim}{r}_{C/B} = \omega_{BC}\,\underset{\sim}{k} \times \left(8\,\underset{\sim}{i} + 2\,\underset{\sim}{j}\right) = \omega_{BC}\left(-2\,\underset{\sim}{i} + 8\,\underset{\sim}{j}\right)$$

Substituting into the relative velocity equation (*) gives the following scalar equations:

$$\boxed{\begin{aligned} -6\omega_{CD} &= -40 - 2\omega_{BC} \\ 2\omega_{CD} &= \;\; 20 + 8\omega_{BC} \end{aligned}} \quad \Rightarrow \quad \boxed{\begin{aligned} 2\omega_{BC} - 6\omega_{CD} &= -40 \\ -8\omega_{BC} + 2\omega_{CD} &= \;\; 20 \end{aligned}}$$

Solving gives

$$\omega_{BC} = -10/11 \approx -0.909 \quad \Rightarrow \boxed{\underset{\sim}{\omega}_{BC} \approx -0.909\,\underset{\sim}{k} \text{ (r/s)}}$$

$$\omega_{CD} = 70/11 \approx 6.36364 \quad \Rightarrow \boxed{\underset{\sim}{\omega}_{CD} \approx 6.36\,\underset{\sim}{k} \text{ (r/s)}}$$

So, in the position shown, link *BC* is rotating ***clockwise*** at a rate of 0.909 (r/s), and link *CD* is rotating ***counterclockwise*** at a rate of 6.36 (r/s).

Note: Here again, the ***signs*** of the variables $\omega_{BC}$ and $\omega_{CD}$ are found by solving the simultaneous equations. So, the directions of the motions of the two links need not be known prior to solving the equations.