Elementary Dynamics Example #35: (Rigid Body Kinematics – Relative Acceleration)

Given: $\ell_1 = \ell_2 = 0.4$ (m)

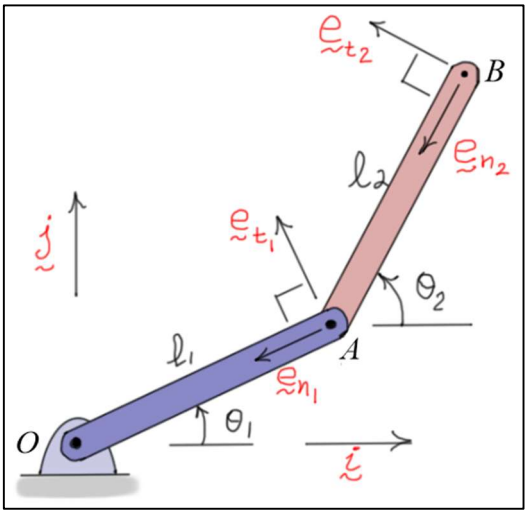
$\theta_1 = 25$ (deg), $\theta_2 = 60$ (deg)

$\omega_1 = \dot{\theta}_1 = 10$ (r/s) (CCW)

$\omega_2 = \dot{\theta}_2 = 5$ (r/s) (CCW)

$\alpha_1 = \dot{\omega}_1 = 20$ (r/s^2) (CCW)

$\alpha_2 = \dot{\omega}_2 = -15$ (r/s^2) (CW)



Find: a_B

Solution:

Using the relative acceleration equation,

$$\boxed{a_B = a_A + a_{B/A}}$$

where

$$\begin{aligned} a_A = a_{A/O} &= \left[\alpha_1 \times r_{A/O}\right] - \left[\omega_1^2 r_{A/O}\right] \\ &= \left[20k \times 0.4\left(\cos(25)\, i + \sin(25)\, j\right)\right] - 10^2\,(0.4)\left(\cos(25)\, i + \sin(25)\, j\right) \\ &= 8\left(-\sin(25)\, i + \cos(25)\, j\right) - 40\left(\cos(25)\, i + \sin(25)\, j\right) \\ &\approx -39.6333\, i - 9.65427\, j \end{aligned}$$

$$\begin{aligned} a_{B/A} &= \left[\alpha_2 \times r_{B/A}\right] - \left[\omega_2^2 r_{B/A}\right] \\ &= \left[-15k \times 0.4\left(\cos(60)\, i + \sin(60)\, j\right)\right] - 5^2\,(0.4)\left(\cos(60)\, i + \sin(60)\, j\right) \\ &= -6\left(-\sin(60)\, i + \cos(60)\, j\right) - 10\left(\cos(60)\, i + \sin(60)\, j\right) \\ &\approx 0.196152\, i - 11.6603\, j \end{aligned}$$

$$\Rightarrow \boxed{a_B \approx -39.4\, i - 21.3\, j} \text{ (m/s}^2\text{)}$$

Notes:

1. The terms $\alpha_1 \times r_{A/O}$ and $\alpha_2 \times r_{B/A}$ are along the e_{t_1} and e_{t_2} directions, respectively.
2. The terms $\left[-\omega_1^2\, r_{A/O}\right]$ and $\left[-\omega_2^2\, r_{B/A}\right]$ are along the e_{n_1} and e_{n_2} directions, respectively.