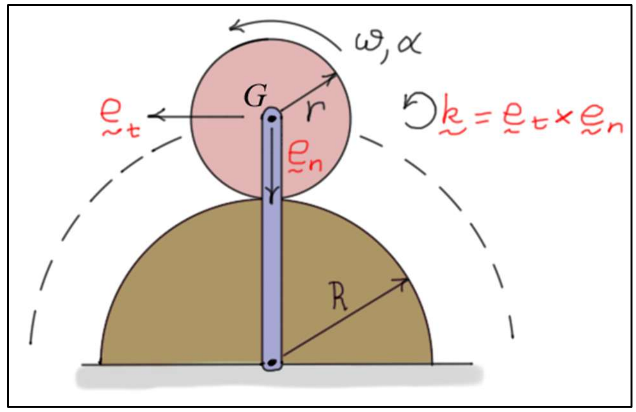
Elementary Dynamics Example #36: (Rigid Body Kinematics – Relative Acceleration)

Given: ω, α, r, R

no slip between the fixed circular surface (sun gear) and the planetary gear

C is the contact point on the planetary gear



Find: $\underset{\sim}{a}_G$, $\underset{\sim}{a}_C$

Solution:

From a previous example on relative velocity,

$$\boxed{\underset{\sim}{v}_G = v\underset{\sim}{e}_t = r\omega\underset{\sim}{e}_t}$$

Using normal and tangential components, the acceleration of G can be written as

$$\boxed{\underset{\sim}{a}_G = \dot{v}\underset{\sim}{e}_t + \left(\frac{v^2}{\rho}\right)\underset{\sim}{e}_n = r\dot{\omega}\underset{\sim}{e}_t + \left(\frac{(r\omega)^2}{R+r}\right)\underset{\sim}{e}_n = r\alpha\underset{\sim}{e}_t + \left(\frac{(r\omega)^2}{R+r}\right)\underset{\sim}{e}_n}$$

Using the relative acceleration equation, the acceleration of C can be written as

$$\boxed{\underset{\sim}{a}_C = \underset{\sim}{a}_G + \underset{\sim}{a}_{C/G}}$$ (C and G are two points on the planetary gear)

where

$$\boxed{\underset{\sim}{a}_{C/G} = \left[\alpha\underset{\sim}{k} \times \underset{\sim}{r}_{C/G}\right] - \omega^2\underset{\sim}{r}_{C/G} = \left[\alpha\underset{\sim}{k} \times r\underset{\sim}{e}_n\right] - \omega^2\left(r\underset{\sim}{e}_n\right) = -r\alpha\underset{\sim}{e}_t - r\omega^2\underset{\sim}{e}_n}$$

Substituting the two results into the relative acceleration equation gives

$$\underset{\sim}{a}_C = \cancel{(r\alpha - r\alpha)}\underset{\sim}{e}_t + \left(\frac{(r\omega)^2}{R+r} - r\omega^2\right)\underset{\sim}{e}_n = \left(\frac{r^2\omega^2 - r\omega^2(R+r)}{R+r}\right)\underset{\sim}{e}_n$$

$$= \left(\frac{\cancel{r^2\omega^2} - \cancel{r^2\omega^2} - rR\omega^2}{R+r}\right)\underset{\sim}{e}_n \qquad \Rightarrow \boxed{\underset{\sim}{a}_C = -\left(\frac{rR}{r+R}\right)\omega^2\underset{\sim}{e}_n}$$

As expected, the acceleration of C the contact point on the moving body (the planetary gear) is normal (perpendicular) to the contacting surfaces.