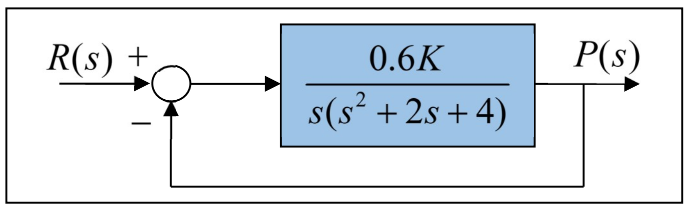
Introductory Control Systems
What is a Root Locus Diagram?

Definition: A ***root locus diagram*** is the plot of the ***paths*** of the poles of a closed loop system as a single parameter K is varied.

Root Locus Diagram:	$0 \leq K < +\infty$
Complementary Root Locus Diagram:	$-\infty < K \leq 0$

Example:

- ***Proportional position control*** of a ***space platform*** can be expressed by the block diagram below. The open-loop platform dynamics is ***second order*** and ***under-damped***.



Question: How do the ***poles*** of this closed-loop system ***vary*** as the parameter K changes?

- One way to answer this question is simply to find the closed loop transfer function, identify the ***characteristic equation***, and compute the ***roots*** for a variety of parameter values. For this system, the characteristic equation is $\boxed{s^3 + 2s^2 + 4s + 0.6K = 0}$.
- The figure below shows a ***plot*** of the ***poles*** of the system for a given set of K values. The red stars indicate the locations of the poles for $1 \leq K \leq 27$ with an increment $\Delta K = 2$. The blue circles indicate the locations of the poles for $-1 \leq K \leq -27$ with an increment of $\Delta K = -2$.

Observations about this root locus diagram:

- As K ***increases*** (becoming more ***positive***), the ***real pole*** moves from the ***origin*** along the ***negative real axis***, while a pair of ***complex conjugate poles*** crosses over the imaginary axis and move into the ***right-half plane***. As K ***decreases*** (becoming more negative), the ***real pole*** moves from the origin along the ***positive real axis***, while a pair of ***complex conjugate poles*** move further into the ***left-half plane***.

- As K increases toward $+\infty$ or decreases toward $-\infty$ the branches follow ***asymptotes***.
- The poles ***do not*** necessarily ***move uniformly*** along the branches.
- Although the poles do not move uniformly along the branches, the branches are ***smooth curves***.
- Recall that if all the poles of a system are in the ***left-half plane***, it is ***stable***, and if any of the poles are in the ***right-half plane***, it is ***unstable***. This system is stable for $0 \leq K \leq 13$ (approximately). A ***more precise estimate*** of the upper limit of K can be found using the Routh-Hurwitz method or by specifying smaller increments in MATLAB.