

Intermediate Dynamics

Lagrange's Equations for Multi-Degree-of-Freedom Systems

The *configuration* of systems with N degrees-of-freedom (DOF) can be defined in terms of N *generalized coordinates*, say q_k ($k = 1, \dots, N$). The *differential equations of motion* of the system can be derived using *Lagrange's equations* as defined in Eq. (1).

$$\boxed{\frac{d}{dt} \left(\frac{\partial K}{\partial \dot{q}_k} \right) - \frac{\partial K}{\partial q_k} = F_{q_k}} \quad (k = 1, \dots, N) \quad (1)$$

Here, (...with NB representing the number of bodies in the system)

$$K = \frac{1}{2} \sum_{j=1}^{NB} \left\{ m_j \left({}^R \underline{v}_{G_j} \right)^2 + {}^R \underline{\omega}_{B_j} \cdot \underline{H}_{G_j} \right\} \quad \dots \text{the } \mathbf{kinetic energy} \text{ of the system}$$

F_{q_k} ...the *generalized force* associated with the *generalized coordinate* q_k
(due to *all* the forces and torques acting on the system)

Note:

It is important that the *kinetic energy* K and the *generalized forces* F_{q_k} ($k = 1, \dots, N$) be written *only in terms* of q_k ($k = 1, \dots, N$), \dot{q}_k ($k = 1, \dots, N$), and *no other variables*.

If some of the forces and torques are *conservative*, their contributions to the equations of motion can be calculated in terms of *potential energy functions*. In this case, the differential equations of motion can be derived using the form of Lagrange's equations given in Eq. (2).

$$\boxed{\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_k} \right) - \frac{\partial L}{\partial q_k} = \left(F_{q_k} \right)_{nc}} \quad (k = 1, \dots, N) \quad (2)$$

Here, $L = K - V$ is the *Lagrangian* of the system, V is the *potential energy function* for the *conservative forces and torques*, and $\left(F_{q_k} \right)_{nc}$ is the *generalized force* associated with q_k for the *nonconservative forces and torques*, only.