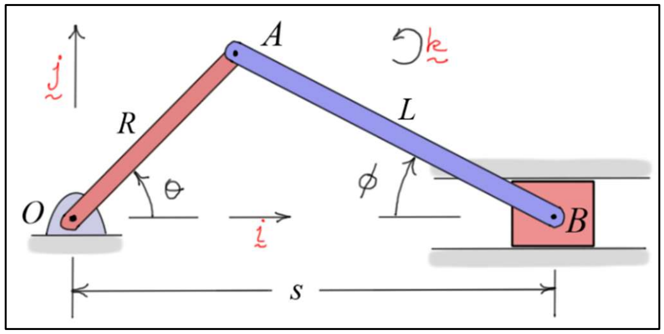
Elementary Dynamics Example #37: (Rigid Body Kinematics – Relative Acceleration)



Given: $R = 3$ (in), $L = 6$ (in), $\theta = 30$ (deg)

$\omega_{OA} = \dot{\theta} = 100$ (rpm) (CCW)

≈ 10.472 (r/s)

$\alpha_{OA} = \dot{\omega}_{OA} = -5\ (\text{r/s}^2)$ (CW)

Find: α_{AB}, $a_B = \ddot{s}$

Solution:

From the triangle formed by the mechanism,

$$\boxed{R\sin(\theta) = L\sin(\phi)} \Rightarrow \boxed{\phi = \sin^{-1}\left(\frac{R\sin(\theta)}{L}\right)_{\theta=30\ (\text{deg})} = 14.4775\ (\text{deg})}$$

Using the ***relative velocity*** equation, the following results were found in Example #32.

$$\boxed{\underset{\sim}{\omega}_{AB} \approx -4.68\,\underset{\sim}{k}\ (\text{rad/s})} \quad \boxed{\underset{\sim}{v}_B \approx -22.7\,\underset{\sim}{i}\ (\text{in/s}) \approx -1.89\,\underset{\sim}{i}\ (\text{ft/s})}$$

Using the ***relative acceleration*** equation, write

$$\boxed{\underset{\sim}{a}_B = \underset{\sim}{a}_A + \underset{\sim}{a}_{B/A}}\ *$$

Here,

$$\boxed{\underset{\sim}{a}_B = a_B\,\underset{\sim}{i}}$$

$$\underset{\sim}{a}_A = \underset{\sim}{a}_{A/O} = \left[\underset{\sim}{\alpha}_{OA} \times \underset{\sim}{r}_{A/O}\right] - \left[\omega_{OA}^2 \underset{\sim}{r}_{A/O}\right]$$

$$= \left[-5\,\underset{\sim}{k} \times 3\left(\cos(30)\,\underset{\sim}{i} + \sin(30)\,\underset{\sim}{j}\right)\right] - 3\omega_{OA}^2\left(\cos(30)\,\underset{\sim}{i} + \sin(30)\,\underset{\sim}{j}\right)$$

$$= -15\left(-\sin(30)\,\underset{\sim}{i} + \cos(30)\,\underset{\sim}{j}\right) - 3\omega_{OA}^2\left(\cos(30)\,\underset{\sim}{i} + \sin(30)\,\underset{\sim}{j}\right)$$

$$\Rightarrow \boxed{\underset{\sim}{a}_A \approx -277.411\,\underset{\sim}{i} - 177.484\,\underset{\sim}{j}}\ (\text{in/s}^2)$$

$$\underset{\sim}{a}_{B/A} = \left[\underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{B/A}\right] - \left[\omega_{AB}^2 \underset{\sim}{r}_{B/A}\right]$$

$$= \left[\alpha_{AB}\,\underset{\sim}{k} \times 6\left(\cos(\phi)\,\underset{\sim}{i} - \sin(\phi)\,\underset{\sim}{j}\right)\right] - 6\omega_{AB}^2\left(\cos(\phi)\,\underset{\sim}{i} - \sin(\phi)\,\underset{\sim}{j}\right)$$

$$= 6\alpha_{AB}\left(\sin(\phi)\,\underset{\sim}{i} + \cos(\phi)\,\underset{\sim}{j}\right) + \left(-127.416\,\underset{\sim}{i} + 32.8985\,\underset{\sim}{j}\right)$$

$$\Rightarrow \boxed{\underset{\sim}{a}_{B/A} \approx \left(6\alpha_{AB}\sin(\phi) - 127.416\right)\underset{\sim}{i} + \left(6\alpha_{AB}\cos(\phi) + 32.8985\right)\underset{\sim}{j}}$$

Substituting back into the relative acceleration equation (*) gives the scalar equations:

$$\boxed{\begin{aligned} a_B &= -277.411 + 6\alpha_{AB}\sin(\phi) - 127.416 \\ 0 &= -177.484 + 6\alpha_{AB}\cos(\phi) + 32.8985 \end{aligned}}$$

Solving the equations: $\Rightarrow$

$$\boxed{\begin{aligned} &\underset{\sim}{\alpha}_{AB} \approx 24.9\,\underset{\sim}{k}\ (\text{r/s}^2) \\ &\underset{\sim}{a}_B \approx -367\,\underset{\sim}{i}\ (\text{in/s}^2) \approx -30.6\,\underset{\sim}{i}\ (\text{ft/s}^2) \end{aligned}}$$