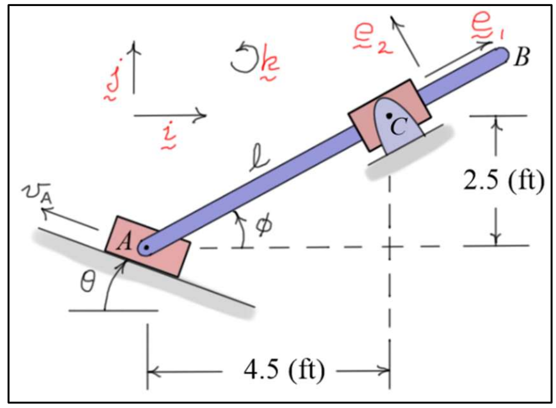
Elementary Dynamics Example #38a: (Rigid Body Kinematics – Sliding Contact Acceleration)

Given: $v_A = 2$ (ft/s) = constant , $\theta = 30$ (deg)

Find: a) ω_{AB} , $\dot{\ell}$, b) α_{AB} , $\ddot{\ell}$



Solution #1: using the unit vectors $\left(\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k}\right)$

a) Using the velocity equation for sliding contacts, write

$$\boxed{\underset{\sim}{v}_C = \underset{\sim}{v}_A + \left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{C/A}\right) + \underset{\sim}{v}_{C_{\text{rel}}}} \; * \quad (C \text{ is fixed, but moves on } AB)$$

Here,

$$\boxed{\underset{\sim}{v}_C = \underset{\sim}{0}} \quad \boxed{\underset{\sim}{v}_A = 2\left(-\cos(20)\,\underset{\sim}{i} + \sin(20)\,\underset{\sim}{j}\right)}$$

$$\boxed{\underset{\sim}{\omega}_{AB} \times \underset{\sim}{r}_{C/A} = \omega_{AB}\,\underset{\sim}{k} \times \left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right) = \omega_{AB}\left(-2.5\,\underset{\sim}{i} + 4.5\,\underset{\sim}{j}\right)}$$

$$\boxed{\underset{\sim}{v}_{C_{\text{rel}}} = \dot{\ell}\,\underset{\sim}{e}_1 = \dot{\ell}\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)\Big/\ell = \dot{\ell}\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)\Big/\sqrt{4.5^2 + 2.5^2}} \quad \text{(velocity of } C \text{ on } AB)$$

Substituting into the velocity equation (*) gives the following scalar equations.

$$\boxed{\begin{aligned} 0 &= -2\cos(20) - 2.5\,\omega_{AB} + \left(\tfrac{4.5}{\ell}\right)\dot{\ell} \\ 0 &= 2\sin(20) + 4.5\,\omega_{AB} + \left(\tfrac{2.5}{\ell}\right)\dot{\ell} \end{aligned}} \quad \ldots\text{solving gives} \Rightarrow \boxed{\begin{cases} \omega_{AB} \approx -0.293458 \approx -0.293 \text{ (r/s)} \\ \dot{\ell} \approx 1.31068 \approx 1.31 \text{ (ft/s)} \end{cases}}$$

b) Using the acceleration equation for sliding contacts, write

$$\boxed{\underset{\sim}{a}_C = \underset{\sim}{a}_A + \left(\underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{C/A}\right) - \omega_{AB}^2\,\underset{\sim}{r}_{C/A} + \underset{\sim}{a}_{C_{\text{rel}}} + 2\left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{v}_{C_{\text{rel}}}\right)} \; ** \quad (C \text{ is fixed, but moves on } AB)$$

Here,

$$\boxed{\underset{\sim}{a}_C = \underset{\sim}{a}_A = \underset{\sim}{0}}$$

$$\boxed{\underset{\sim}{\alpha}_{AB} \times \underset{\sim}{r}_{C/A} = \alpha_{AB}\,\underset{\sim}{k} \times \left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right) = \alpha_{AB}\left(-2.5\,\underset{\sim}{i} + 4.5\,\underset{\sim}{j}\right)} \quad \boxed{-\omega_{AB}^2\,\underset{\sim}{r}_{C/A} = -\omega_{AB}^2\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)}$$

$$\boxed{\underset{\sim}{a}_{C_{\text{rel}}} = \ddot{\ell}\,\underset{\sim}{e}_1 = \ddot{\ell}\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)\Big/\ell = \ddot{\ell}\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)\Big/\sqrt{4.5^2 + 2.5^2}} \quad \text{(acceleration of } C \text{ on } AB)$$

$$\boxed{2\left(\underset{\sim}{\omega}_{AB} \times \underset{\sim}{v}_{C_{\text{rel}}}\right) = 2\omega_{AB}\,\underset{\sim}{k} \times \dot{\ell}\left(4.5\,\underset{\sim}{i} + 2.5\,\underset{\sim}{j}\right)\Big/\ell = \left(\frac{2\omega_{AB}\dot{\ell}}{\ell}\right)\left(-2.5\,\underset{\sim}{i} + 4.5\,\underset{\sim}{j}\right)}$$

Substituting into the acceleration equation (**) gives the following scalar equations.

$$\boxed{\begin{aligned} 0 &= -2.5\,\alpha_{AB} - 4.5\,\omega_{AB}^2 + \left(\tfrac{4.5}{\ell}\right)\ddot{\ell} - \left(\frac{5\omega_{AB}\dot{\ell}}{\ell}\right) \\ 0 &= 4.5\,\alpha_{AB} - 2.5\,\omega_{AB}^2 + \left(\tfrac{2.5}{\ell}\right)\ddot{\ell} + \left(\frac{9\omega_{AB}\dot{\ell}}{\ell}\right) \end{aligned}} \quad \ldots\text{solving gives} \Rightarrow \boxed{\begin{cases} \alpha_{AB} \approx 0.149434 \approx 0.149 \text{ (r/s}^2) \\ \ddot{\ell} \approx 0.443318 \approx 0.443 \text{ (ft/s}^2) \end{cases}}$$

Conclusions:

1. Bar AB is rotating clockwise $(\omega_{AB} < 0)$, but slowing down $(\alpha_{AB} > 0)$.
2. Point C is moving away from point A $(\dot{\ell} > 0)$ at an increasing rate $(\ddot{\ell} > 0)$.