

Elementary Dynamics Example #38b: (Rigid Body Kinematics – Sliding Contact Acceleration)

Given: $v_A = 2$ (ft/s) = constant, $\theta = 30$ (deg)

Find: a) ω_{AB} , $\dot{\ell}$, b) α_{AB} , $\ddot{\ell}$

Solution #2: using the unit vectors (e_1, e_2, k)

a) Using the velocity equation for sliding contacts, write (C is fixed, but moves on AB)

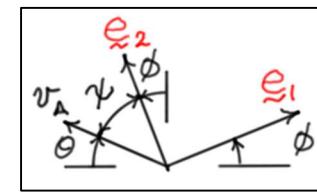
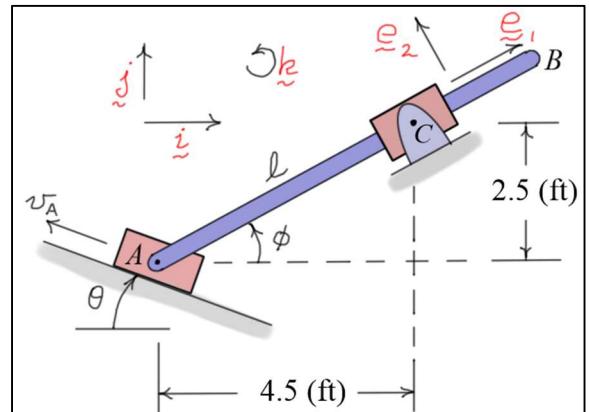
$$\underline{v}_C = \underline{v}_A + (\omega_{AB} \times \underline{r}_{C/A}) + \underline{v}_{C_{\text{rel}}} \quad ***$$

Here,

$$\underline{v}_C = 0 \quad \omega_{AB} \times \underline{r}_{C/A} = \omega_{AB} k \times \ell e_1 = \ell \omega_{AB} e_2$$

$$\underline{v}_{C_{\text{rel}}} = \dot{\ell} e_1 \quad (\text{velocity of } C \text{ on } AB)$$

$$\underline{v}_A = 2(-\sin(\psi) e_1 + \cos(\psi) e_2) \quad \psi = 90 - \phi - \theta \approx 40.9454 \text{ (deg)}$$



Substituting into the velocity equation (****) gives the following scalar equations.

$$\begin{aligned} 0 &= -2 \sin(\psi) + \dot{\ell} \\ 0 &= 2 \cos(\psi) + \ell \omega_{AB} \end{aligned} \quad \text{...solving gives} \Rightarrow \begin{cases} \omega_{AB} \approx -0.293458 \approx -0.293 \text{ (r/s)} \\ \dot{\ell} \approx 1.31068 \approx 1.31 \text{ (ft/s)} \end{cases}$$

b) Using the acceleration equation for sliding contacts, write

$$\underline{a}_C = \underline{a}_A + (\alpha_{AB} \times \underline{r}_{C/A}) - \omega_{AB}^2 \underline{r}_{C/A} + \underline{a}_{C_{\text{rel}}} + 2(\omega_{AB} \times \underline{v}_{C_{\text{rel}}}) \quad **** \quad (C \text{ is fixed, but moves on } AB)$$

where

$$\underline{a}_C = \underline{a}_A = 0 \quad \alpha_{AB} \times \underline{r}_{C/A} = \alpha_{AB} k \times \ell e_1 = \ell \alpha_{AB} e_2 \quad -\omega_{AB}^2 \underline{r}_{C/A} = -\ell \omega_{AB}^2 e_1$$

$$\underline{a}_{C_{\text{rel}}} = \ddot{\ell} e_1 \quad (\text{acceleration of } C \text{ on } AB) \quad 2(\omega_{AB} \times \underline{v}_{C_{\text{rel}}}) = 2\omega_{AB} k \times \dot{\ell} e_1 = 2\omega_{AB} \dot{\ell} e_2$$

Substituting into the acceleration equation (****) gives the following scalar equations.

$$\begin{aligned} 0 &= -\ell \omega_{AB}^2 + \ddot{\ell} \\ 0 &= \ell \alpha_{AB} + 2\dot{\ell} \omega_{AB} \end{aligned} \quad \text{...solving gives} \Rightarrow \begin{cases} \alpha_{AB} \approx 0.149434 \approx 0.149 \text{ (r/s}^2\text{)} \\ \ddot{\ell} \approx 0.443318 \approx 0.443 \text{ (ft/s}^2\text{)} \end{cases}$$

Conclusions:

1. As expected, the results are the **same** as in Example #38a. The results **do not depend** on the unit vectors used to express the equations.
2. The unit vectors chosen for this second solution produce a **less complicated set** of scalar equations than those in Example #38a. A **careful selection** of unit vectors (directions) can often make the solution process **less tedious**.