

Elementary Dynamics Example #38b: (Rigid Body Kinematics – Sliding Contact Acceleration)

Given: $v_A = 2 \text{ (ft/s)} = \text{constant}$, $\theta = 30 \text{ (deg)}$

Find: a) ω_{AB} , $\dot{\ell}$, b) α_{AB} , $\ddot{\ell}$

Solution #2: using the unit vectors ($\underline{e}_1, \underline{e}_2, \underline{k}$)

a) Using the velocity equation for sliding contacts, write (C is fixed, but moves on AB)

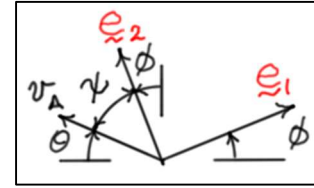
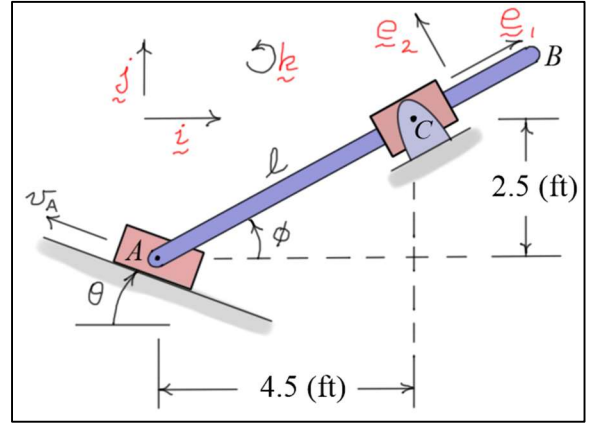
$$\underline{v}_C = \underline{v}_A + (\underline{\omega}_{AB} \times \underline{r}_{C/A}) + \underline{v}_{C_{\text{rel}}} \quad ***$$

Here,

$$\underline{v}_C = \underline{0} \quad \underline{\omega}_{AB} \times \underline{r}_{C/A} = \omega_{AB} \underline{k} \times \ell \underline{e}_1 = \ell \omega_{AB} \underline{e}_2$$

$$\underline{v}_{C_{\text{rel}}} = \dot{\ell} \underline{e}_1 \quad (\text{velocity of C on AB})$$

$$\underline{v}_A = 2(-\sin(\psi) \underline{e}_1 + \cos(\psi) \underline{e}_2) \quad \psi = 90 - \phi - \theta \approx 40.9454 \text{ (deg)}$$



Substituting into the velocity equation (***) gives the following scalar equations.

$$\begin{cases} 0 = -2\sin(\psi) + \dot{\ell} \\ 0 = 2\cos(\psi) + \ell \omega_{AB} \end{cases} \quad \dots \text{solving gives} \Rightarrow \begin{cases} \omega_{AB} \approx -0.293458 \approx -0.293 \text{ (r/s)} \\ \dot{\ell} \approx 1.31068 \approx 1.31 \text{ (ft/s)} \end{cases}$$

b) Using the acceleration equation for sliding contacts, write

$$\underline{a}_C = \underline{a}_A + (\underline{\alpha}_{AB} \times \underline{r}_{C/A}) - \omega_{AB}^2 \underline{r}_{C/A} + \underline{a}_{C_{\text{rel}}} + 2(\underline{\omega}_{AB} \times \underline{v}_{C_{\text{rel}}}) \quad **** \quad (C \text{ is fixed, but moves on AB})$$

where

$$\underline{a}_C = \underline{a}_A = \underline{0} \quad \underline{\alpha}_{AB} \times \underline{r}_{C/A} = \alpha_{AB} \underline{k} \times \ell \underline{e}_1 = \ell \alpha_{AB} \underline{e}_2 \quad -\omega_{AB}^2 \underline{r}_{C/A} = -\ell \omega_{AB}^2 \underline{e}_1$$

$$\underline{a}_{C_{\text{rel}}} = \ddot{\ell} \underline{e}_1 \quad (\text{acceleration of C on AB}) \quad 2(\underline{\omega}_{AB} \times \underline{v}_{C_{\text{rel}}}) = 2\omega_{AB} \underline{k} \times \dot{\ell} \underline{e}_1 = 2\omega_{AB} \dot{\ell} \underline{e}_2$$

Substituting into the acceleration equation (****) gives the following scalar equations.

$$\begin{cases} 0 = -\ell \omega_{AB}^2 + \ddot{\ell} \\ 0 = \ell \alpha_{AB} + 2\dot{\ell} \omega_{AB} \end{cases} \quad \dots \text{solving gives} \Rightarrow \begin{cases} \alpha_{AB} \approx 0.149434 \approx 0.149 \text{ (r/s}^2\text{)} \\ \ddot{\ell} \approx 0.443318 \approx 0.443 \text{ (ft/s}^2\text{)} \end{cases}$$

Conclusions:

1. As expected, the results are the **same** as in Example #38a. The results **do not depend** on the unit vectors used to express the equations.
2. The unit vectors chosen for this second solution produce a **less complicated set** of scalar equations than those in Example #38a. A **careful selection** of unit vectors (directions) can often make the solution process **less tedious**.