

Elementary Dynamics Example #39: (Rigid Body Kinetics – Mass Center/Inertia)

Given: $m_{AB} = 2.4 \text{ (kg)}$, $m_{CD} = 4.5 \text{ (kg)}$, $\rho_{\text{plate}} = 12 \text{ (kg/m}^2\text{)}$

$r_o = 0.3 \text{ (m)}$, $r_i = 0.1 \text{ (m)}$, G is the composite mass center

the circular plate is thin and the rods (AB and CD) are slender

Find: a) $\bar{y}$; b) I_G ; c) I_O

Solution:

a) To find the location of G relative to the top bar AB , write

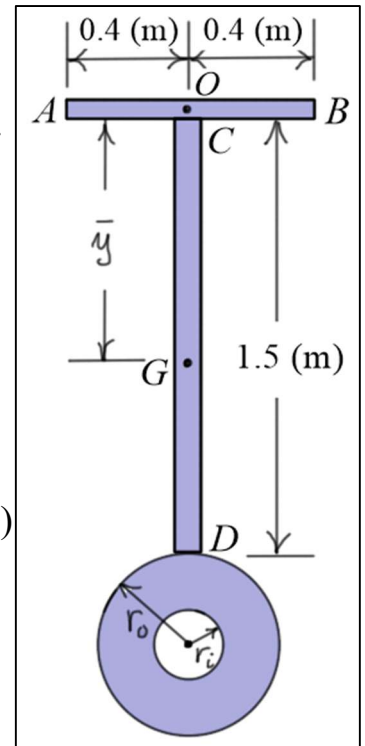
$$m_{AB}(0) + m_{CD}(0.75) + m_{\text{plate}}(1.8) = (m_{AB} + m_{CD} + m_{\text{plate}})\bar{y}$$

where

$$m_{\text{plate}} = \rho A = \rho \pi (r_o^2 - r_i^2) \approx 3.01593 \text{ (kg)} \quad (\text{circular plate with hole})$$

So, the distance to the mass center is

$$\bar{y} = \left[\frac{m_{AB}(0) + m_{CD}(0.75) + m_{\text{plate}}(1.8)}{(m_{AB} + m_{CD} + m_{\text{plate}})} \right] \approx 0.887831 \text{ (m)}$$



b) The inertia of the composite shape about the composite mass center G is the sum of the inertias of the components about G .

$$I_G = (I_G)_{AB} + (I_G)_{CD} + (I_G)_{\text{plate}} \quad *$$

Using the parallel axes theorem for each of the components gives

$$(I_G)_{AB} = \frac{1}{12} m_{AB} (0.8)^2 + m_{AB} \bar{y}^2 \approx 2.01978 \text{ (kg-m}^2\text{)}$$

$$(I_G)_{CD} = \frac{1}{12} m_{CD} (1.5)^2 + m_{CD} (\bar{y} - 0.75)^2 \approx 0.929239 \text{ (kg-m}^2\text{)}$$

$$m_{\text{full plate}} = \rho A = \rho \pi r_o^2 \approx 3.39292 \text{ (kg)} \quad m_{\text{hole}} = \rho A = \rho \pi r_i^2 \approx 0.376991 \text{ (kg)}$$

$$(I_G)_{\text{plate}} = \frac{1}{2} m_{\text{full plate}} r_o^2 - \frac{1}{2} m_{\text{hole}} r_i^2 + m_{\text{plate}} (1.8 - \bar{y})^2 \approx 2.66021 \text{ (kg-m}^2\text{)}$$

Substituting into the inertia equation (*) gives

$$I_G \approx 5.60923 \approx 5.61 \text{ (kg-m}^2\text{)}$$

c) Using the parallel axes theorem on the composite shape, write

$$I_O = I_G + m_{\text{total}} \bar{y}^2 \approx 5.60923 + \left[(2.4 + 4.5 + 3.01593)(0.887831)^2 \right] \approx 13.4254 \approx 13.4 \text{ (kg-m}^2\text{)}$$