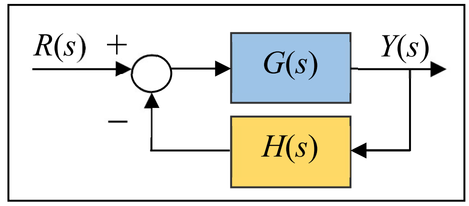
Introductory Control Systems
Summary Procedure for the Root Locus Diagram



Simple Closed Loop System

1. Write the characteristic equation: $\boxed{1 + GH(s) = 0}$

   Rewrite the equation in the form: $\boxed{1 + kP(s) = 0}$

   <u>Root Locus (RL)</u>: $0 \le k \le +\infty$

2. Find the ***poles*** and ***zeros*** of $P(s)$. The root loci start at the poles and proceed to the zeros as k advances from $0 \to \infty$.
   - The number of branches (loci) on the RL diagram is equal to n_p, the number of poles.
   - The number of asymptotes is $n_A = n_p - n_z$. (n_z is the number of zeros)

3. Plot the ***pole-zero diagram*** for $P(s)$. Then,
   - Identify those segments of the real axis that contain roots of the characteristic equation. There are roots on those segments such that an ***odd*** number of poles and zeros are to the right of that segment.
   - Identify the direction of movement of the poles. The poles of the closed loop system move from the poles of $P(s)$ to the zeros of $P(s)$ (or to infinity) as K increases from $0 \to \infty$.

4. Calculate the ***angles of all asymptotes*** (if any):

$$\boxed{\phi_A = \left[\frac{2m+1}{n_p - n_z}\right] 180^o} \quad \left(m = 0, 1, 2, \ldots, (n_p - n_z - 1)\right)$$

5. Calculate the ***intersection point*** of the asymptotes with the real axis (if any):

$$\boxed{\sigma_A = \frac{\sum(\text{pole locations}) - \sum(\text{zero locations})}{n_p - n_z}}$$

6. ***Sketch*** the branches of the root locus diagram. Keep in mind that the root loci are ***symmetric*** with respect to the ***real axis***.

7. Calculate the locations of the ***break points*** (if any):

   Define: $\boxed{p(s) = -1/P(s)}$ Set: $\boxed{\frac{dp(s)}{ds} = 0 \quad \textbf{or} \quad \frac{dP(s)}{ds} = 0}$ and solve for s.

8. ***Angles of Departure*** and ***Arrival*** of the Root Loci:
   - The angle of the tangent to the root locus at any point must satisfy the angle condition: The ***difference*** between the sum of the angles of the vectors drawn to the point from the poles of $P(s)$ and the sum of the angles of the vectors drawn to it from the zeros of $P(s)$ is an ***odd multiple of 180^o***.
   - At a pole of $P(s)$, the angle is called an angle of departure. At a zero of $P(s)$, the angle is called an angle of arrival.