

Elementary Dynamics Example #42: (Rigid Body Kinetics – Fixed Axis Rotation)

Given: $W = 10$ (lb), $\ell = 2$ (ft), $\theta = 30$ (deg), $\omega = 5$ k (r/s)

Find: α , the angular acceleration of the bar

reaction forces on the pin at O

Solution:

Applying Newton's laws of motion to the free-body-diagram in the normal and tangential directions and taking moments about the fixed-point O , write

$$\uparrow \sum F_n = O_n - W \cos(\theta) = ma_n = m\left(\frac{1}{2}\ell\omega^2\right)$$

$$\Rightarrow O_n = 10 \cos(30) + \frac{1}{2} \frac{10}{g} (5^2) = 16.4242 \approx 16.4 \text{ (lb)}$$

$$\nearrow \sum F_t = O_t - W \sin(\theta) = ma_t = m\left(\frac{1}{2}\ell\alpha\right)$$

$$\curvearrowright \sum M_O = -\left(\frac{1}{2}\ell \sin(\theta)\right)W = I_O \alpha \quad I_O = \frac{1}{3}m\ell^2 = \frac{1}{3}\frac{W}{g}2^2 = \frac{4W}{3g}$$

Solving the moment equation for the angular acceleration gives

$$\alpha = -\frac{W\ell \sin(\theta)}{2I_O} = -\frac{(W\ell \sin(\theta))(3g)}{2(4W)} = -\frac{(2 \sin(30))(3g)}{8}$$

$$= -12.075 \approx -12.1 \text{ (r/s}^2\text{)}$$

Substituting this result into the tangential force equation gives

$$O_t = W \sin(\theta) + m\left(\frac{1}{2}\ell\alpha\right) = (10 \sin(30)) + \frac{1}{2} \frac{10}{g} (-12.1) \approx 1.25 \text{ (lb)}$$

Alternatively, the moment equation could be written about the mass-center G . This results in a **second equation** for O_t and α that can be solved **simultaneously** with the tangential force equation.

$$\curvearrowright \sum M_G = -\left(\frac{1}{2}\ell\right)O_t = I_G \alpha = \left(\frac{1}{12}m\ell^2\right)\alpha = \frac{1}{12}\left(\frac{W}{g}\right)2^2\alpha = \left(\frac{W}{3g}\right)\alpha$$

Simultaneous equations:

$$\begin{cases} O_t - \left(\frac{W}{g}\right)\alpha = W \sin(30) = 5 \\ O_t + \left(\frac{W}{3g}\right)\alpha = 0 \end{cases} \Rightarrow \begin{cases} O_t \approx 1.25 \text{ (lb)} \\ \alpha \approx -12.1 \text{ (r/s}^2\text{)} \end{cases}$$

