

Elementary Dynamics Example #43: (Rigid Body Kinetics – General Plane Motion #1)

Given: $r_o = 0.4$ (m), $r_i = 0.25$ (m), $m = 100$ (kg)
 $P = 200$ (N), $\theta = 20$ (deg), $k_G = 0.3$ (m)
 $\mu_s = 0.2$, $\mu_k = 0.15$

Find: α , the angular acceleration of the spool S

Solution:

There are two possibilities – either the spool **slips**, or it **does not slip**.

Assumption: S **rolls without slipping** on the horizontal plane

In this case, the acceleration of G is directly related to the angular acceleration of S .

$$\underline{a}_G = r_o \alpha \underline{i} \quad (\text{assuming that } \alpha \text{ is positive clockwise})$$

Using the free-body-diagram and Newton's laws of motion, write

$$\rightarrow \sum F_x = P \cos(20) - f = ma_G = mr_o \alpha$$

$$\uparrow \sum F_y = N + P \sin(20) - mg = 0 \Rightarrow N = mg - P \sin(20) = 182.519 \approx 183 \text{ (N)}$$

$$\curvearrowright \sum M_G = r_o f - r_i P = I_G \alpha = mk_G^2 \alpha$$

The first and third equations can be solved simultaneously for f and α .

$$\begin{cases} f + (100 \times 0.4) \alpha = P \cos(20) = 200 \cos(20) \\ 0.4 f - (100 \times 0.3^2) \alpha = r_i P = 50 \end{cases} \Rightarrow \begin{cases} f = 147.658 \approx 148 \text{ (N)} \\ \alpha = 1.00702 \approx 1.00 \text{ (r/s}^2\text{)} \end{cases}$$

Check: The signs of the friction force and the angular acceleration are consistent with the assumption. Also, $f \approx 148 \text{ (N)} < f_{\max} \approx 183 \text{ (N)}$, so the spool does not slip.

Notes:

- If the spool slips, $f = \mu_k N$ and $a_G \neq r_o \alpha$. In this case, the **friction** and **normal forces** are **directly related**, but the **acceleration** of G and the **angular acceleration** of the spool are **not**. If this assumption is made, you should find the conditions of the assumption are **violated**. You may want to try this!
- Some find the above results **troubling**. But, in fact, the spool does **roll** to the **right**. If the angle θ is increased to 90 (deg), the spool will **roll** to the **left**. It is not difficult to find the angle θ_t for which the spool will not roll. In this case, it simply **translates** to the right. (Ans: $\theta_t \approx 47.6$ (deg))

