

Elementary Dynamics Example #45a: (Rigid Body Kinetics – Work and Energy #1)

Given: $\ell = 4$ (m), $m = 18$ (kg), $M = 50$ (N-m) = constant
released from rest at $\theta = 10$ (deg), neglect friction

Find: ω the angular velocity of AB when $\theta = 40$ (deg)

Solution: position 1 is $\theta = 10$ (deg), and position 2 is $\theta = 40$ (deg)

$$KE_1 + U_{1 \rightarrow 2} = KE_2$$

Here,

$$KE_1 = 0 \quad (\text{released from rest})$$

$$KE_2 = \frac{1}{2} I_{IC} \omega^2 = \frac{1}{2} \left(\frac{1}{3} m \ell^2 \right) \omega^2 = \frac{1}{6} m \ell^2 \omega^2 = 48 \omega^2$$

$$\begin{aligned} U_{1 \rightarrow 2} &= (U_{1 \rightarrow 2})_{\text{gravity}} + (U_{1 \rightarrow 2})_M = V_1 - V_2 + M(\Delta\theta) \\ &= mg\left(\frac{\ell}{2}\right)\cos(10) - mg\left(\frac{\ell}{2}\right)\cos(40) + 50(40 - 10)\frac{\pi}{180} \\ &= 77.2585 + 26.1799 \\ &\Rightarrow U_{1 \rightarrow 2} = 103.438 \text{ (N-m)} \end{aligned}$$

Solving,

$$48\omega^2 = 103.438 \Rightarrow \omega \approx 1.47 \text{ (r/s)} \quad \curvearrowright$$

Notes:

1. The **instantaneous center (IC)** is used to calculate the kinetic energy of AB . Compare this to using the **general formula** $KE = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$. To use the general formula in this case, the mass-center velocity v_G must be written in terms of the angular velocity ω .
2. This problem becomes much more difficult if **friction** is included. Why?

