

### Elementary Dynamics Example #45b: (Rigid Body Kinetics – Work and Energy #2)

Given:  $r_o = 0.4$  (m),  $r_i = 0.25$  (m),  $m = 100$  (kg)

$P = 200$  (N),  $\theta = 20$  (deg),  $k_G = 0.3$  (m)

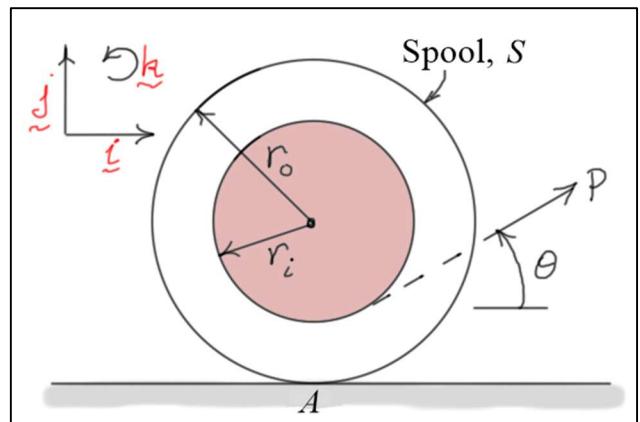
spool rolls without slipping

released from rest

Find:  $\omega$  the angular velocity of  $S$  after  $G$  moves 2 (m) to the right.

Solution: ( $A$  is the instantaneous center of the spool)

$$KE_1 + U_{1 \rightarrow 2} = KE_2 \quad (\text{initial state: 1, final state: 2})$$



Here,

$$KE_1 = 0 \quad (\text{released from rest})$$

$$KE_2 = \frac{1}{2} I_A \omega_2^2 = \frac{1}{2} (I_G + m r_o^2) \omega_2^2 = \frac{1}{2} (m k_G^2 + m r_o^2) \omega_2^2 = 12.5 \omega_2^2$$

The spool **rolls without slipping**, so the only force that does work is  $P$ . The **distance**  $d_P$  over which  $P$  acts can be calculated as follows.

$$d_P = \underbrace{\Delta x_G \cos(\theta)}_{\text{translation of } S} - \underbrace{r_i \Delta \theta}_{\text{rotation of } S} = 2 \cos(\theta) - r_i \left( \frac{\Delta x_G}{r_o} \right) = 2 \cos(20^\circ) - 0.25 \left( \frac{2}{0.4} \right) \approx 0.629385 \text{ (m)}$$

Using this result, the **work done** by  $P$  can be calculated as follows.

$$U_{1 \rightarrow 2} = P d_P \approx 200 (0.629385) \approx 125.877 \text{ (N-m)}$$

Substituting into the work-energy equation gives:  $\omega_2 \approx \sqrt{\frac{125.877}{12.5}} \approx 3.17335 \approx 3.17 \text{ (rad/s)}$ .

As the spool rolls to the **right**, the angular motion is **clock-wise**.

Check:

From an **earlier analysis** using **Newton's law**, the **angular acceleration** of the spool was calculated to be  $\alpha \approx 1.00702 \text{ (rad/s}^2\text{)}$ . Given a **constant angular acceleration**, the change in angular velocity can be calculated as follows.

$$\omega_2^2 = \omega_1^2 + 2\alpha \Delta \theta = 2\alpha \left( \frac{\Delta x_G}{r_o} \right) \approx 2(1.00702) \left( \frac{2}{0.4} \right) \Rightarrow \omega_2 \approx \sqrt{10.0702} \approx 3.17 \text{ (rad/s)}$$

Question:

It is **assumed** here that the spool **rolls without slipping**. How can this assumption be **verified**?  
Answer: Newton's law. See Example #43.