Elementary Dynamics Example #46: (Rigid Body Kinetics – Conservation of Energy)

Given: AB and BC are slender rods, $\ell = 3$ (ft)

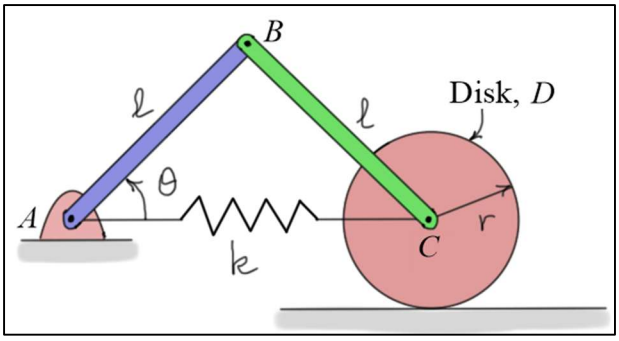
$W_{AB} = W_{BC} = W = 15$ (lb), $W_D = 20$ (lb)

$k = 4$ (lb/ft)

spring is unstretched at $\theta = 45$ (deg)

released from rest at $\theta = 45$ (deg)

Disk rolls without slipping, $r = 1$ (ft)



Find: ω_{AB} the angular velocity of AB at $\theta = 0$ (deg)

Solution:

Using the ***conservation of energy*** on the system (state 1: $\theta = 45$ (deg); state 2: $\theta = 0$ (deg))

$$\underbrace{K_1}_{\text{zero}} + V_1 = K_2 + V_2 \quad \text{(released from rest)}$$

Defining a datum at the horizontal line through A and C gives

$$V_1 = (V_1)_{\text{gravity}} + \underbrace{(V_1)_{\text{spring}}}_{\text{zero}} = 2\left(W\tfrac{\ell}{2}\sin(45)\right) \approx 31.8198 \text{ (ft-lb)}$$

$$V_2 = \underbrace{(V_2)_{\text{gravity}}}_{\text{zero}} + (V_2)_{\text{spring}} = \tfrac{1}{2}k\left(2\ell - 2\ell\cos(45)\right)^2 = 2k\ell^2\left(1-\cos(45)\right)^2 \approx 6.17662 \text{ (ft-lb)}$$

In state 2, the disk has ***zero velocity***, AB is rotating about A, and the ***instantaneous center*** of BC is at C. Also, due to the ***symmetry*** of the mechanism, the angular velocities of AB and BC are ***equal*** (but in opposite directions).

$$K_2 = (K_2)_{\text{disk}} + (K_2)_{AB} + (K_2)_{BC} = \underbrace{0}_{(v_C)_2 = 0} + \tfrac{1}{2}I_A\omega_{AB}^2 + \tfrac{1}{2}I_C\omega_{BC}^2$$

$$= \tfrac{1}{2}\left(\tfrac{1}{3}\left(\tfrac{W}{g}\right)\ell^2\right)\omega_{AB}^2 + \tfrac{1}{2}\left(\tfrac{1}{3}\left(\tfrac{W}{g}\right)\ell^2\right)\omega_{AB}^2 = \tfrac{1}{3}\left(\tfrac{W}{g}\right)\ell^2\omega_{AB}^2 \approx 1.39752\,\omega_{AB}^2$$

Substituting into the conservation of energy equation gives

$$31.8198 = 6.17662 + 1.39752\,\omega_{AB}^2 \quad \Rightarrow \quad \omega_{AB} = \sqrt{\frac{31.8198 - 6.17662}{1.39752}} \approx 4.28 \text{ (rad/s)}$$

The motion of AB is ***clockwise***, and the motion of BC is ***counter-clockwise***.

Question: What other forces (besides the spring and gravity) affect the motion of this system? Why is the work done by these forces on the system equal to zero?