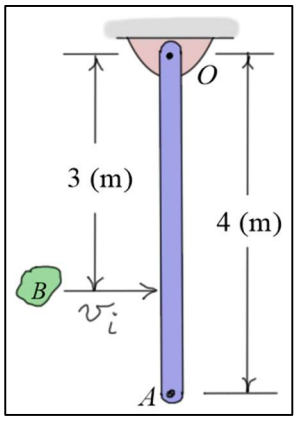
Elementary Dynamics Example #50: (Rigid Body Kinetics – Impulse & Momentum – Impact #1)



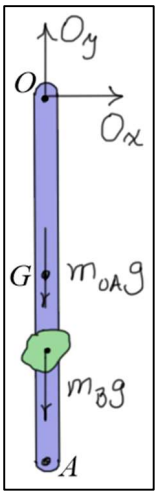
Given: bar OA is initially at rest when the blob B strikes it
B sticks to the bar and they move together after impact
$m_{OA} = 15$ (kg); $m_B = 5$ (kg); $v_i = 10$ (m/s)

Find: $\theta_{\max}$, the maximum angle the bar reaches after impact

Solution:

During the impact, $\sum M_O = 0$, so the ***angular momentum*** of the ***system*** about O is ***conserved***.

$\boxed{\curvearrowleft (H_O)_1 = (H_O)_2}$ (state 1: just before impact; state 2: just after impact)



Here,

$$\boxed{(H_O)_1 = 3(m_B v_i) = 3\times 5\times 10 = 150 \text{ (N-m-s)}}$$

$$\boxed{\begin{aligned}(H_O)_2 &= 3(m_B v_B)_2 + I_O\omega_2 = 3m_B(3\omega_2) + \left(\tfrac{1}{3}m_{OA}\ell^2\right)\omega_2 = \left(45 + \left(\tfrac{1}{3}\times 15\times 4^2\right)\right)\omega_2 \\ &= 125\omega_2\end{aligned}}$$

Substituting into the conservation of angular momentum equation gives

$\boxed{\omega_2 = 150/125 = 1.2 \text{ (rad/s)}}$ (angular velocity of OA just after impact)

To find the ***maximum angle*** the bar reaches, apply the ***conservation of energy***. Defining the ***datum*** of the ***bar*** at the bottom-most position of G, and the ***datum*** of ***B*** at its bottom-most position and observing the ***kinetic energy*** of the system is ***zero*** at the ***maximum angle***, write

$\boxed{KE_2 + \underbrace{V_2}_{\text{zero}} = \underbrace{KE_3}_{\text{zero}} + V_3}$ (state 2: just after impact; state 3: at maximum angle)

Here,

$$\boxed{\begin{aligned}KE_2 &= \tfrac{1}{2}I_O\omega_2^2 + \tfrac{1}{2}m_B\left(v_B^2\right)_2 = \tfrac{1}{2}\left(\tfrac{1}{3}m_{OA}\ell^2\right)\omega_2^2 + \tfrac{1}{2}m_B(3\omega_2)^2 = \tfrac{1}{2}(80)(1.2)^2 + \tfrac{9}{2}(5)(1.2)^2 \\ &= 90 \text{ (N-m)}\end{aligned}}$$

$$\boxed{\begin{aligned}V_3 &= m_{OA}g\left(2 - 2\cos(\theta_{\max})\right) + m_B g\left(3 - 3\cos(\theta_{\max})\right) = \left(2m_{OA} + 3m_B\right)g\left(1 - \cos(\theta_{\max})\right) \\ &= 45g\left(1 - \cos(\theta_{\max})\right)\end{aligned}}$$

Substituting and solving:

$$45g\left(1 - \cos(\theta_{\max})\right) = 90 \quad \Rightarrow 1 - \cos(\theta_{\max}) = 90/(45g) \quad \Rightarrow \theta_{\max} = \cos^{-1}\left(\frac{45g - 90}{45g}\right)$$

$$\Rightarrow \boxed{\theta_{\max} \approx 37.24 \text{ (deg)}}$$