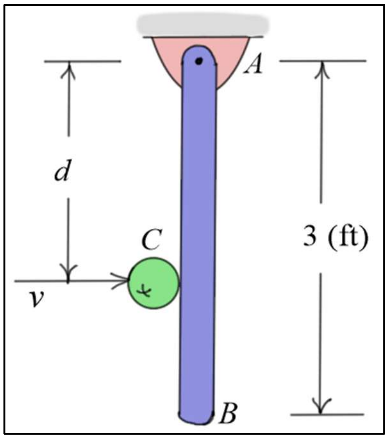
Elementary Dynamics Example #51: (Rigid Body Kinetics – Impulse & Momentum – Impact #2)



Given: bar AB is initially at rest when the ball C strikes it

$W_{AB} = 6$ (lb) ; $W_C = 1$ (lb) ; $e = 0.7$; $d = 2$ (ft)

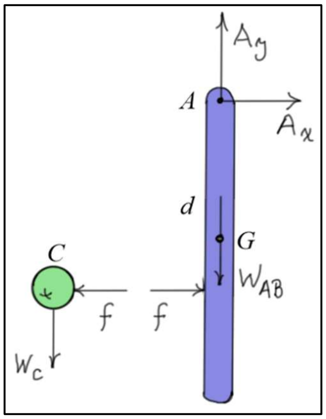
just before impact, the velocity of C is $v = 50$ (ft/s)

Find: angular velocity of AB and velocity of C just after impact

Solution: (system defined as bar AB and ball C)

The ***angular impulse*** on the system during the impact is ***zero***. So, the ***angular momentum*** of the system about A is ***conserved*** during the impact. In the analysis that follows, the ***weight forces*** are assumed to be ***non-impulsive***.

$$\circlearrowleft\; \left(\underset{\sim}{H}_A\right)_1 = \left(\underset{\sim}{H}_A\right)_2$$ (state 1: just before impact; state 2: just after impact)



$$\left(\underset{\sim}{H}_A\right)_1 = \left(\frac{W_C}{g}\right)(v_C)_1 d = \frac{1\times 50\times 2}{32.2} \approx 3.10559 \text{ (ft-lb-s)}$$

$$\left(\underset{\sim}{H}_A\right)_2 = \left(\frac{W_C}{g}\right)(v_C)_2 d + I_A\left(\omega_{AB}\right)_2 \quad\Rightarrow\quad \left(\underset{\sim}{H}_A\right)_2 = \left(\tfrac{2}{g}\right)(v_C)_2 + \left(\tfrac{18}{g}\right)\left(\omega_{AB}\right)_2$$

Substituting into the ***conservation of angular momentum*** equation gives

$$2(v_C)_2 + 18\left(\omega_{AB}\right)_2 = 3.10559\times g = 100$$

The ***impact*** equation is

$$e = \frac{(v_{\hat{C}})_2 - (v_C)_2}{(v_C)_1 - (v_{\hat{C}})_1} = \frac{d\left(\omega_{AB}\right)_2 - (v_C)_2}{50-0} \quad\Rightarrow\quad -(v_C)_2 + 2\left(\omega_{AB}\right)_2 = 50e = 35$$

Solving the last two boxed equations ***simultaneously*** gives

$$(v_C)_2 \approx -19.5455 \approx -19.5 \text{ (ft/s)}$$
$$\left(\omega_{AB}\right)_2 \approx 7.72727 \approx 7.73 \text{ (rad/s)}$$

The ball rebounds to the ***left***, and the bar ***rotates counter-clockwise*** at the rates shown.