

Elementary Engineering Mathematics

Equation Sheet #2 – Two Dimensional Vectors & Complex Numbers

2D Vectors

1. Vector Magnitude and Direction

$$|\underline{V}| = \sqrt{V_x^2 + V_y^2} \quad \theta = \tan^{-1}(V_y/V_x)$$

2. Vector Sum

$$\underline{V}_1 + \underline{V}_2 + \dots + \underline{V}_n = (V_{1x} + V_{2x} + \dots + V_{nx})\underline{i} + (V_{1y} + V_{2y} + \dots + V_{ny})\underline{j}$$

3. Scalar (Dot) Product of Two Vectors

$$\underline{A} \cdot \underline{B} = |\underline{A}||\underline{B}|\cos(\underline{A}, \underline{B}) = (a_x \underline{i} + a_y \underline{j}) \cdot (b_x \underline{i} + b_y \underline{j}) = a_x b_x + a_y b_y$$

4. Vector Components Parallel and Perpendicular to a Unit Vector $\underline{n}$

$$\underline{A}_{\parallel} = (\underline{A} \cdot \underline{n})\underline{n} \quad \text{and} \quad \underline{A}_{\perp} = \underline{A} - \underline{A}_{\parallel}$$

5. Vector (Cross) Product of Two Vectors

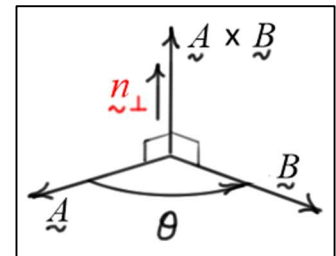
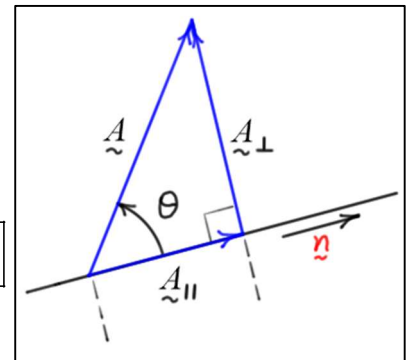
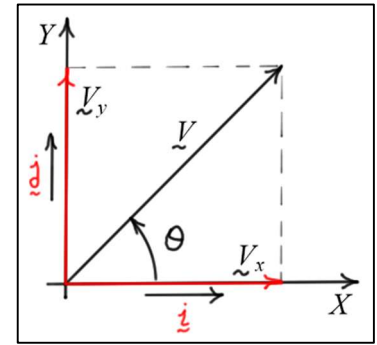
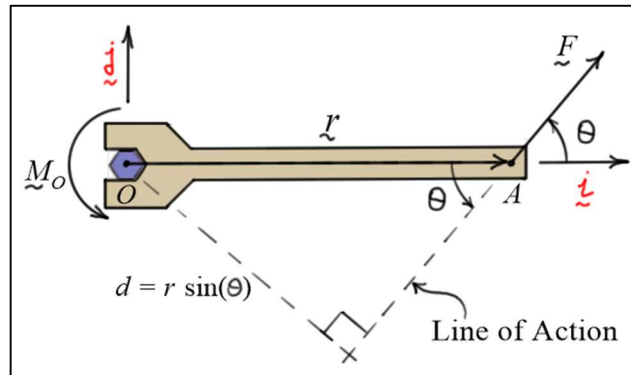
$$\underline{A} \times \underline{B} = (|\underline{A}||\underline{B}|\sin(\underline{A}, \underline{B}))\underline{n}_{\perp} = (a_x \underline{i} + a_y \underline{j}) \times (b_x \underline{i} + b_y \underline{j}) = (a_x b_y - a_y b_x)\underline{k}$$

$$\underline{A} \times \underline{B} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ a_x & a_y & 0 \\ b_x & b_y & 0 \end{vmatrix} = (a_x b_y - a_y b_x)\underline{k}$$

6. Moment of a Force

$$\underline{M}_O = \underline{r} \times \underline{F}$$

$$d = \frac{|\underline{M}_O|}{|\underline{F}|}$$



Complex Numbers ($j = \sqrt{-1}$)

1. Magnitude and Direction

$$A = x + jy = r e^{j\theta} = r \angle \theta \quad |A| = r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}(y/x) \quad e^{j\theta} = \cos(\theta) + j \sin(\theta)$$

2. Sum

$$A_1 + A_2 + \dots + A_n = (x_1 + x_2 + \dots + x_n) + j(y_1 + y_2 + \dots + y_n)$$

3. Products and Ratios

$$A \cdot B = (|A|e^{j\alpha})(|B|e^{j\beta}) = |A||B|e^{j(\alpha+\beta)} \quad A/B = (|A|e^{j\alpha})/(|B|e^{j\beta}) = (|A|/|B|)e^{j(\alpha-\beta)}$$

