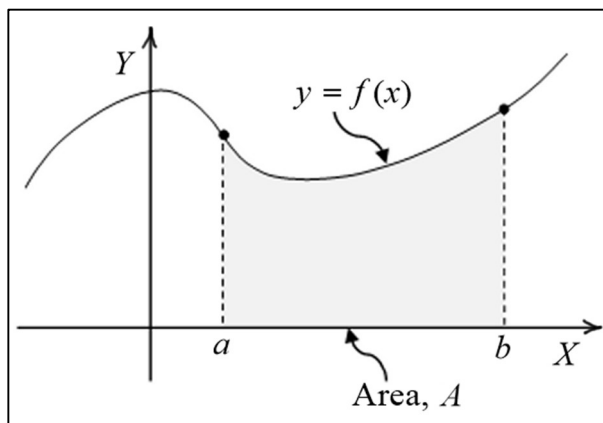# Elementary Engineering Mathematics
Equations Sheet #7 – Integrals/Integration

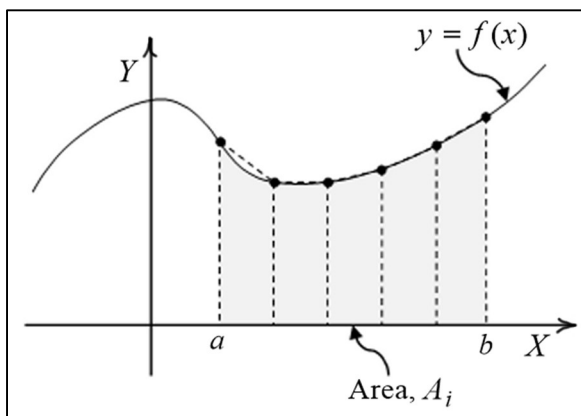1. Definition of a Definite Integral

$$A = \int_a^b f(x)dx$$

2. Approximation of a Definite Integral

$$A \approx \sum_i A_i$$

$$\approx \sum_i \left[\frac{f(x_{i+1}) + f(x_i)}{2} \right] \Delta x_i$$

3. Some Basic Properties of Definite Integrals

	Property	Comment
1	$\int_a^b -f(x)dx = -\int_a^b f(x)dx$	function values have opposite sign, so, areas will also
2	$\int_b^a f(x)dx = -\int_a^b f(x)dx$	increments have opposite sign, so, areas will also
3	$\int_a^a f(x)dx = 0$	width of area = zero
4	$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$	total area = the sum of the areas
5	$\int_a^b \alpha f(x)dx = \alpha \int_a^b f(x)dx$	α is a constant
6	$\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$	integral of a sum = the sum of the integrals

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Equations Sheet #7 – Integrals/Integration (continued)

3. Anti-derivatives of Common Engineering Functions

Name	Function, $f(x)$	Antiderivative, $G(x)$ $(G'(x)=f(x))$
Constant	a	ax
Polynomial terms	ax^n	$ax^{n+1}/(n+1)$
Exponential	e^{ax}	e^{ax}/a
Sine	$\sin(ax)$	$-\cos(ax)/a$
Cosine	$\cos(ax)$	$\sin(ax)/a$

4. Fundamental Theorem of Integral Calculus:

$$\int_a^b f(x)dx = \int_a^b G'(x)dx = G(x)\big|_a^b = G(b) - G(a)$$

5. Integrals as Functions:

$$\int_a^x f(x)dx = \int_a^x G'(x)dx = G(x)\big|_a^x = G(x) - G(a)$$

$$\int f(x)dx = G(x) + C \quad \text{(indefinite integral)}$$