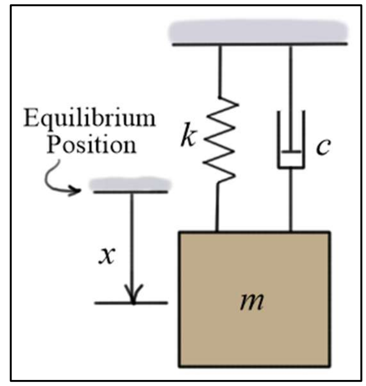
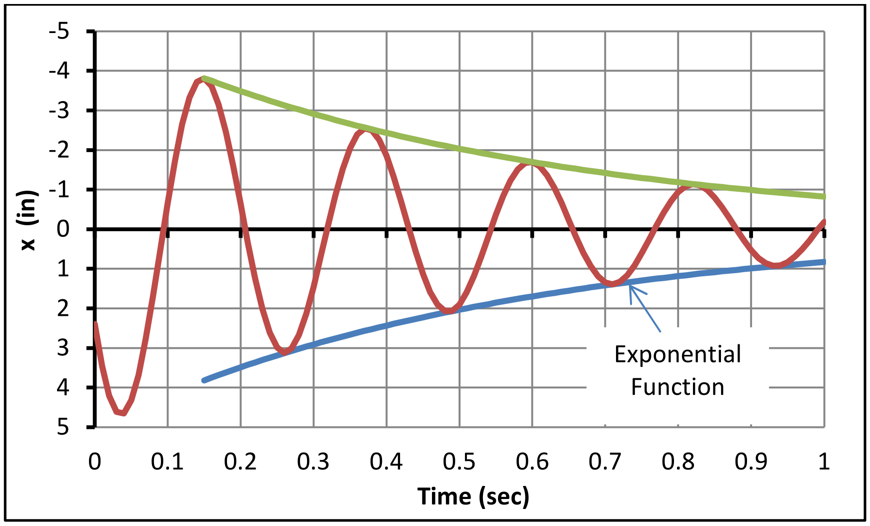
Elementary Engineering Mathematics
Exercises #7 – Exponential, Natural Logarithms, and Trigonometric Functions



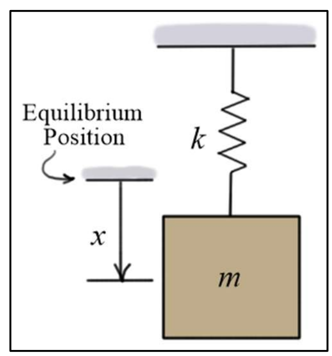
1. The displacement x of a spring-mass-damper system over a one second interval is shown by the red line in the plot below. The downward displacement is bounded by an exponential function shown in blue. The displacements measured at its five bottom-most positions are shown in the table.



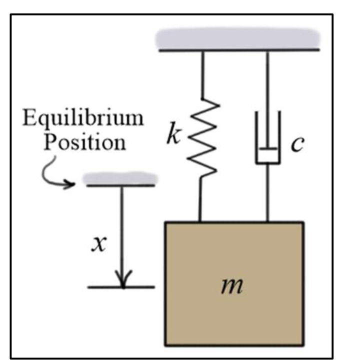
Time, t (sec)	Displacement, x (in)
0.26	3.12
0.48	2.07
0.71	1.39
0.93	0.925

Using the data provided, estimate (a) the frequency of the oscillation ω in (rad/sec), (b) the decay rate of the oscillations α. Assume the displacement is bounded by an exponential function $x = A e^{\alpha t}$.

2. A spring-mass system with $m = 0.4$ (slugs), $k = 100$ (lb/ft), and no damping is shown in the diagram. The system is given an initial displacement of $x_0 = 0.5$ (ft) and initial velocity of $v_0 = 10$ (ft/s). Find (a) $x(t)$ as a sum of sine and cosine functions, (b) $x(t)$ as a single, phase shifted sine function, (c) the time when the mass first reaches its largest displacement, and d) T the period of the oscillation.

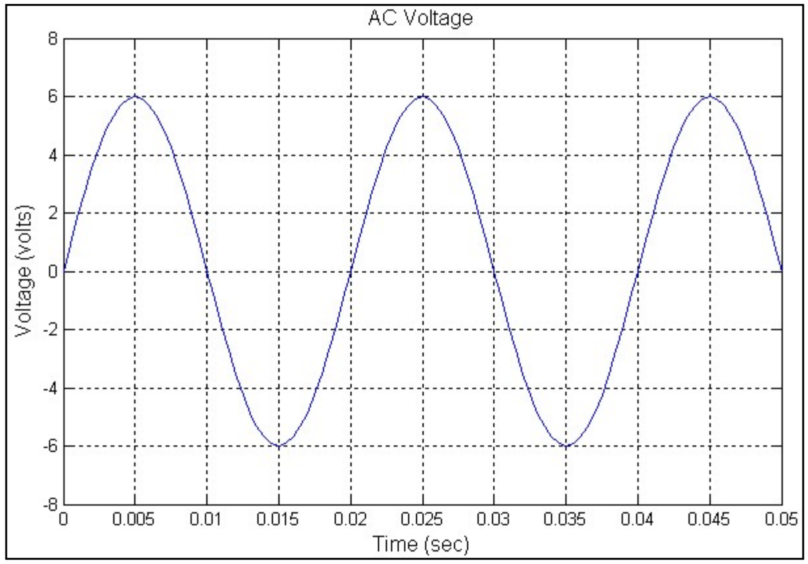


3. A spring-mass-damper system with $m = 0.4$ (slugs), $k = 100$ (lb/ft), and $c = 3$ (lb-s/ft) is shown in the diagram. The system is given an initial displacement $x_0 = 0.5$ (ft) and initial velocity $v_0 = 10$ (ft/s). (a) Calculate c_c the critical damping coefficient, (b) Find the displacement function $x(t)$. Express $x(t)$ as an exponential function times a single, phase-shifted sine function.



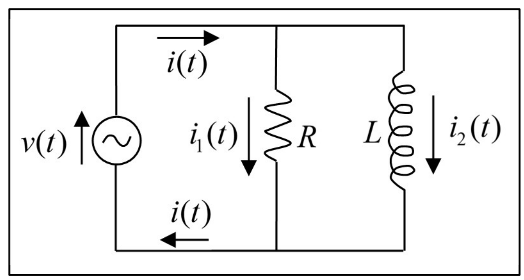
4. A spring-mass-damper system with $m = 0.4$ (slugs), $k = 100$ (lb/ft), and $c = 20$ (lb-s/ft) is shown in the diagram. The system is given an initial displacement $x_0 = 0.5$ (ft) and initial velocity $v_0 = 10$ (ft/s). (a) Calculate c_c the critical damping coefficient, (b) Find the displacement function $x(t)$.

5. For the AC voltage shown, identify the following on the graph.
   a) Period T in seconds
   b) Frequency f in Hertz (cycles/sec)
   c) Frequency ω in radians/sec
   d) Amplitude A



6. In the parallel RL circuit, the applied voltage is $v(t) = 110\cos(120\pi t)$. The currents passing through the resistor and the inductor are given by the expressions

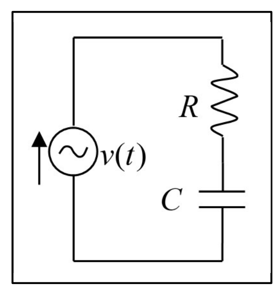
$$i_1(t) = 3\cos(120\pi t)$$
$$i_2(t) = 4\sin(120\pi t)$$



Given that $\boxed{i(t) = i_1(t) + i_2(t)}$, find $i(t)$. Express $i(t)$ in the form of a single, phase-shifted cosine function.

7. In the series RC circuit, the voltages across the resistor and capacitor are given by the expressions

$$v_R(t) = 92\cos(120\pi t + 63.52^\circ)$$
$$v_C(t) = 61\cos(120\pi t - 26.48^\circ)$$



Given that $\boxed{v(t) = v_R(t) + v_C(t)}$, find $v(t)$. Express $v(t)$ in the form of a single, phase-shifted cosine function.