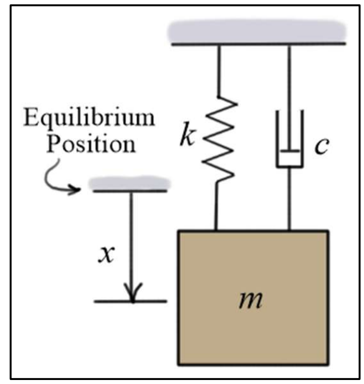
Elementary Engineering Mathematics
Exercises #9 – Derivatives

1. For the under-damped spring-mass-damper system shown, the spring stiffness is $k = 25$ (lb/ft), the damping coefficient is $c = 3$ (lb-s/ft), the mass is $m = 0.25$ (slug), the initial position is $x_0 = 0$ (ft), and the initial velocity is $v_0 = 15$ (ft/s). Using the table of derivatives and the rules for differentiation, differentiate the displacement function to find (a) $v(t) = \dot{x}(t)$ the velocity, and (b) $a(t) = \ddot{x}(t)$ the acceleration of the mass. Using these results, find (c) a_0 the initial acceleration of the mass, and (d) the time when the displacement first reaches a maximum.

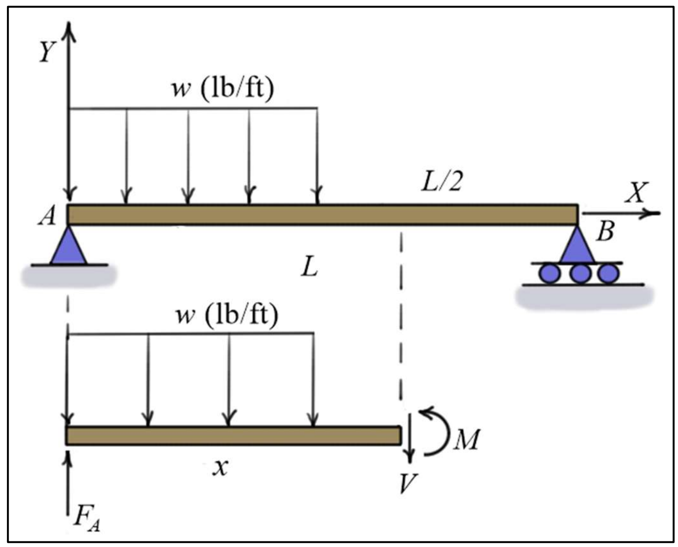


2. The simply supported beam shown has a uniformly distributed load over the left half of the beam. For a beam of length $L = 10$ (ft) and a load $w = 100$ (lb/ft), the internal bending moment is

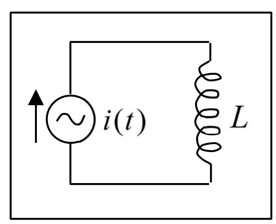
$$\boxed{M(x) = 375x - 50x^2 \text{ (ft-lb)}} \quad \left(0 \le x \le \tfrac{L}{2}\right)$$

$$\boxed{M(x) = 1250 - 125x \text{ (ft-lb)}} \quad \left(\tfrac{L}{2} \le x \le L\right)$$

(a) Find $V(x) = M'(x)$ the shearing force as a function of *x*, (b) Find $\hat{x}$ the location of the maximum bending moment, and (c) Sketch the shearing force and bending moment diagrams. Is the shearing force continuous at $x = L/2$?



3. In the simple circuit shown, the current $i(t) = t^3 e^{-2t}$ (amps), the voltage across the inductor $v(t) = L\dfrac{di}{dt}$, and $L = 125$ (mh). a) Find $v(t)$ the voltage across inductor, b) Find the values of the current when the voltage is zero, and c) Use the above information to sketch $i(t)$. Identify the times on the graph where $v(t) = 0$.



4. In the simple circuit shown, $C = 12$ (μf) and the applied voltage $v(t) = 22e^{-30t}\cos(120\pi t)$ (volts). Find the current $i(t)$. Express the result as an exponential function times a single, phase-shifted cosine function.