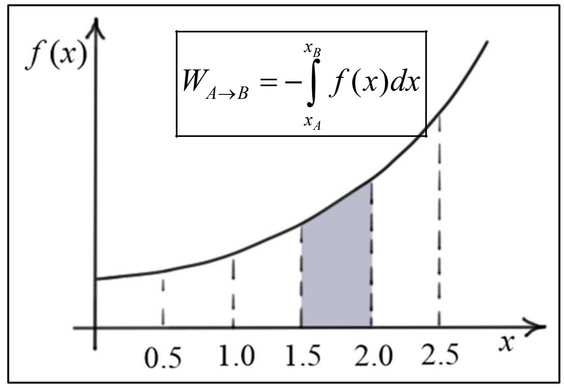
ENGR 1990 Engineering Mathematics
Exercises #10 – Integrals

1. A hardening spring has the force-displacement function $f(x) = 100 + 10x + x^2$ (lb). The work done by the spring as it is stretched over some displacement interval is the negative of the integral of the force-displacement function over that interval. Estimate the integral and the work done by the spring as it is stretched from $x = 0$ to $x = 2.5$ inches by breaking the area into a sequence of trapezoids of width $\Delta x = 0.5$ (in).

x	$f(x)$	Interval	f_{avg}	Δx
0		--		
0.5		1		0.5
1		2		0.5
1.5		3		0.5
2		4		0.5
2.5		5		0.5
		Σ		

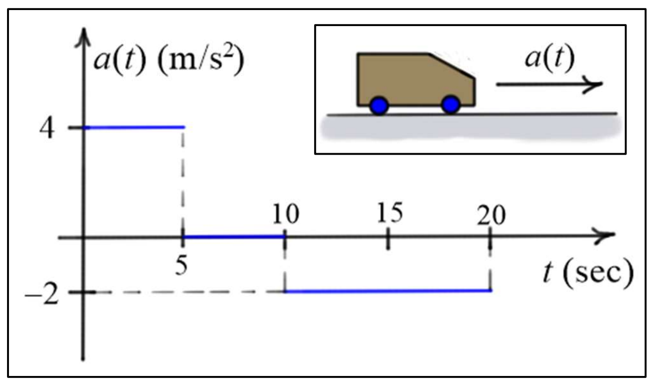


2. Using the same spring force-displacement function given in problem (1), find the work done by the spring using anti-derivatives. Calculate the percent error of the estimate found in problem (1).

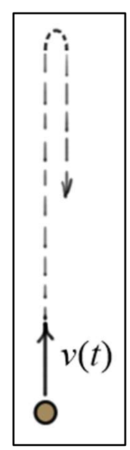
3. A car has the acceleration profile shown, and its initial position and velocity are zero. Given that

$$\boxed{v(t) = \int a(t)dt} \quad \text{and} \quad \boxed{s(t) = \int v(t)dt},$$

find (a) the velocity function $v(t)$, (b) the displacement function $s(t)$, and (c) the total distance traveled by the car for $0 \le t \le 20$ (sec). Sketch the functions.



4. A ball that is thrown upward has velocity $\boxed{v(t) = 75 - 32.2t \text{ (ft/s)}}$. Given that the displacement function of the ball is $\boxed{y(t) = \int v(t)\,dt}$, find (a) the displacement of the ball from its original position after 3.5 seconds, and (b) the total distance traveled by the ball during the 3.5 second period.



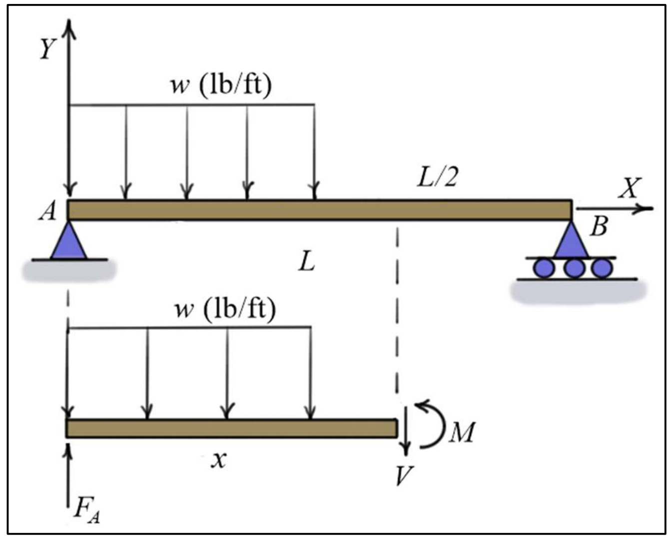
5. A ball is thrown upward with an initial velocity of $v_0 = 20$ (m/s) from an initial height of $y_0 = 8$ (m) and has a constant downward acceleration of $a_0 = -9.81$ (m/s^2). Given that $\boxed{v(t) = \int a(t)dt}$ and $\boxed{y(t) = \int v(t)dt}$, find (a) the velocity function $v(t)$, and (b) the position function $y(t)$.

6. The simply supported beam has a uniformly distributed load over the left half of the beam. For a beam of length $L = 10$ (ft) and a load $w = 100$ (lb/ft), the internal shearing force is

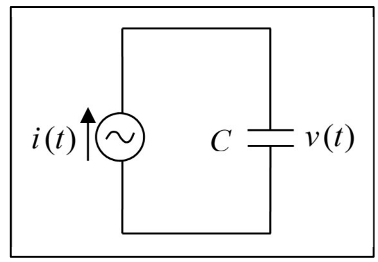
$\boxed{V(x) = 375 - 100\,x \text{ (lb)}}$ $\quad 0 \le x \le 5$ (ft)

$\boxed{V(x) = -125 \text{ (lb)}}$ $\quad 5 \le x \le 10$ (ft)

Given that the internal bending moments at A and B are zero, find $M(x) = \int V(x)dx$ the internal bending moment as a function of x.



7. A current $i(t) = 2\,e^{-2t}$ (amps) is applied to a capacitor with capacitance $C = 0.25$ (f). Given that $v(t) = \frac{1}{C}\int i(t)\,dt$, find the



a) voltage $v(t)$ assuming $v(0) = 0$

b) power, $p(t) = v(t)\cdot i(t)$

c) total energy, $W(t) = \int_0^t p(t)\,dt$ (joules)

What is the energy at $t = 1,2,3,4$ (sec)? What is the limit of the energy as $t \to \infty$?

8. A voltage $v(t) = 10\,\sin(120\,\pi\,t)$ (volts) is applied to an inductor with inductance $L = 250$ (mh). Find the current $i(t)$, given that $i(t) = \frac{1}{L}\int v(t)\,dt$. Assume $i(0) = 0$.