**Elementary Dynamics**
**Exercises #3 – Curvilinear Motion: Nonstationary Unit Vectors**

1. A car **starts** at A with **velocity** $v_A = 4$ (m/s) and **increases** its speed at a rate $\dot{v}(s) = 5 - 0.01s$ (m/s²) as it travels from A to D along a circular track. The angle $\theta = 120$ (deg), and the radius of the path is $R = 100$ (m). Find a) a_t the **tangential component** of the acceleration of the car at D, b) $v(s)$ the velocity of the car as a function of s the distance traveled along the circular path, c) a_n the **normal component** of the acceleration of the car at D, and d) $\dot{\theta}$ and $\ddot{\theta}$ the first and second time derivatives of the angle θ at D.
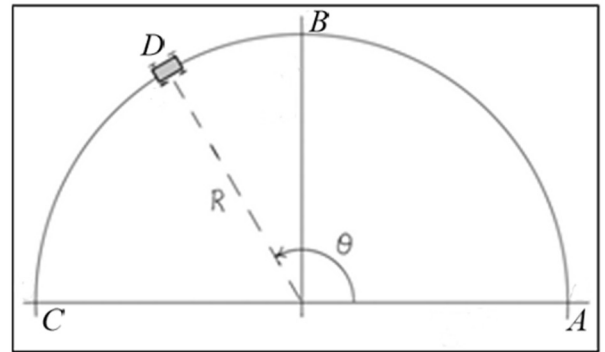
   Answers:

   a) $a_t \approx 2.91$ (m/s²); b) $v(s) = \sqrt{16 + 10\,s - \frac{s^2}{100}}$ (m/s); c) $a_n \approx 16.7$ (m/s²);

   d) $\dot{\theta} \approx 0.409$ (r/s); $\ddot{\theta} \approx 0.0291$ (r/s²)

2. A car is travelling along a road from A to B as shown. The car has speed $v_A = 16$ (m/s) at A and changes speed at the rate $\dot{v}(s) = 3 - 0.02\,s$ (m/s²) as it travels to B. Find:

   a) $v(s)$ the **speed** of the car as a **function** of distance s travelled along the road, and b) a_t and a_n the **tangential** and **normal** components of the **acceleration** of the car at B. The distance along the road from A to B is 120 (m), and the **radius of curvature** of the path at B is $\rho = 100$ (m).
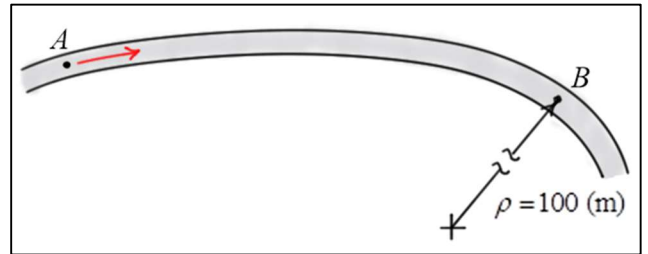
   Answers:

   a) $v(s) = \sqrt{256 - 6\,s - 0.02\,s^2}$ (m/s); b) $a_t = 0.6$ (m/s²), $a_n = 6.88$ (m/s²)

3. As the telescopic robotic arm moves, end A moves so the distance from O to A is given by the **function** $r(\theta) = 1 + 0.5\cos(\theta)$ (m). At the instant when $\theta = \pi/3$ (rad), the arm is rotating **clockwise** with $\dot{\theta} = 0.6$ (rad/s) and $\ddot{\theta} = 0.25$ (rad/s^2). Find: a) v_r and v_θ the **radial** and **transverse components** of the velocity of A at this instant, b) the **angle** ϕ between $\underset{\sim}{e}_\theta$ the **transverse** unit vector and $\underset{\sim}{e}_t$ the **tangential** unit vector, and c) a_r and a_θ the **radial** and **transverse components** of the acceleration of A at this instant.

$r(\theta) = 1 + 0.5\cos(\theta)$

Answers:

a) $v_r = -0.260$ (m/s) , $v_\theta = 0.75$ (m/s); b) $\phi = 19.1$ (deg) $= 0.333$ (rad)

c) $a_r = -0.648$ (m/s^2) , $a_\theta = 7.31 \times 10^{-4}$ (m/s^2)
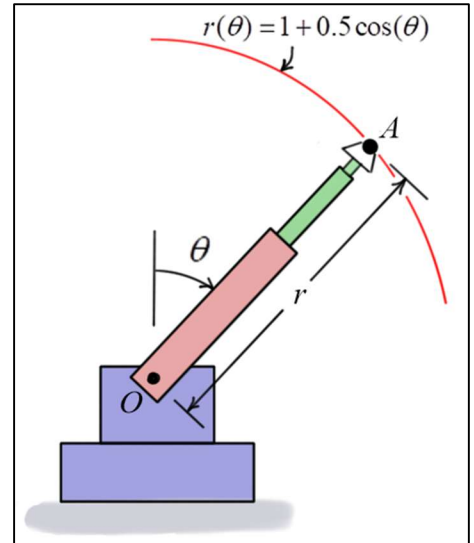
4. Car A is approaching a highway on a circular entrance ramp of radius $R = 200$ (ft). It starts from O at a speed of $v_O = 30$ (ft/sec) and **increases** its speed at a rate $\dot{v}_A = 0.02\,s$ (ft/sec^2). The distance s is measured in feet from point O. Find: a) $v(s)$ the speed of car A as a function of s the distance traveled along the circular path, and b) a_n and a_t the **normal** and **tangential** components of $\underset{\sim}{a}_A$ the acceleration of car A when $\theta = 60$ (deg).
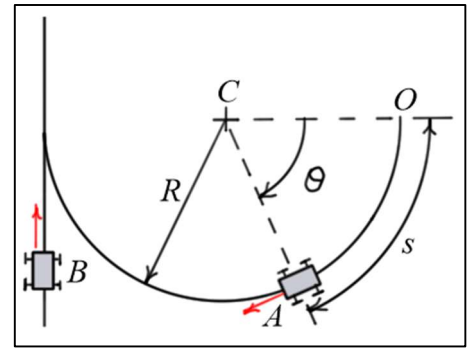

Answers: a) $v(s) = \sqrt{30^2 + 0.02\,s^2}$ (ft/sec); b) $a_n = 8.89$ (ft/sec^2) and $a_t = 4.19$ (ft/sec^2)

5. The slotted arm AB drives the pin C through the spiral groove described by the equation $r = 1.5\theta$ (ft), where θ is in radians. The arm **starts from rest** at $\theta = 60° = \frac{\pi}{3}$ (rad) and is driven at an angular velocity of $\dot{\theta} = 4t$ (rad/s), where time t is in seconds. Find: a) $\theta(t)$ the angle of arm AB as a function of time, b) v_r and v_θ the **radial** and **transverse** components of the **velocity** of C when $t = 0.75$ (sec), and c) a_r and a_θ the **radial** and **transverse** components of the **acceleration** of C when $t = 0.75$ (sec).
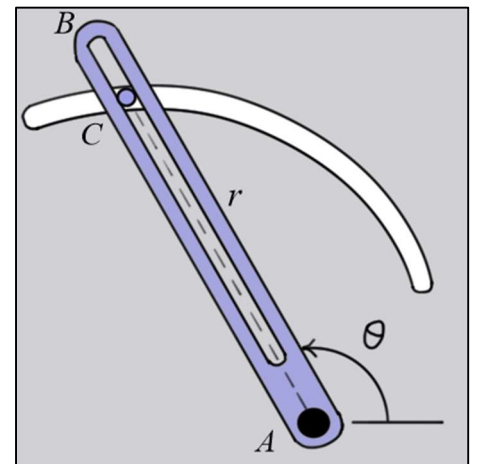

Answers: a) $\theta(t) = 2t^2 + \frac{\pi}{3}$ (rad); b) $v_r = 4.5$ (ft/s); $v_\theta = 9.77$ (ft/s); c) $a_r = -23.3$ (ft/s^2); $a_\theta = 40.0$ (ft/s^2)

6. Bar *OA* rotates **counterclockwise** at a **constant** angular velocity of $\dot{\theta} = 5$ (r/s). The two pin-connected collars at *B* slide freely on *OA* and on the curved rod whose shape is described by the equation $r(\theta) = 100(2 - \cos\theta)$ (mm). When $\theta = 120$ degrees, find: a) v_r and v_θ the **radial** and **transverse** components of $\underset{\sim}{v}_B$ the **velocity** of *B*, b) ϕ the angle between the **radial** direction and the **tangent** to the curved path, and c) a_r and a_θ the **radial** and **transverse** components of $\underset{\sim}{a}_B$ the acceleration of *B*.
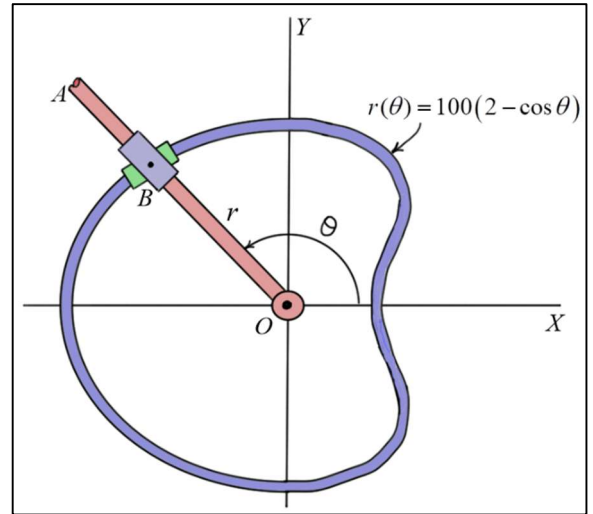
Answers:

a) $v_r = 433$ (mm/s); $v_\theta = 1250$ (mm/s); b) $\phi = 70.9$ (deg); c) $a_r = -7500$ (mm/s^2); $a_\theta = 4330$ (mm/s^2)

7. As the slotted arm *OA* rotates, the pin *P* slides along the surface of the cam. The surface of the cam is defined by the equation $r(\theta) = 2 + \sin(\theta)$ (ft). Given $\theta = 60$ (deg), $\dot{\theta} = 2$ (rad/s), and $\ddot{\theta} = 5$ (rad/s^2), find: a) v_r and v_θ the **radial** and **transverse** components of $\underset{\sim}{v}_P$ the velocity of pin *P*, and b) a_r and a_θ the **radial** and **transverse** components of $\underset{\sim}{a}_P$ the acceleration of pin *P*.
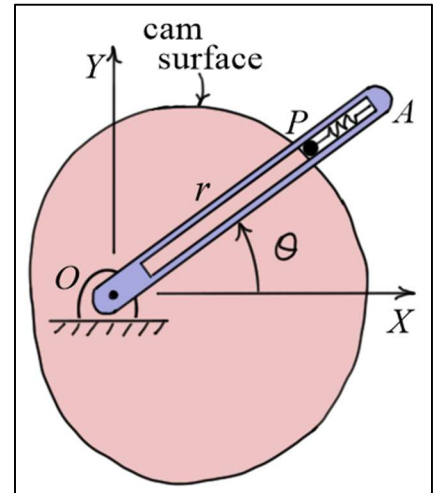
Answers: a) $v_r = 1$ (ft/s); $v_\theta \approx 5.73$ (ft/s);

b) $a_r \approx -12.4$ (ft/s^2); $a_\theta \approx 18.3$ (ft/s^2)