Introductory Control Systems
Exercises #6 – Transfer Functions from Closed Loop Equations

Sketch a ***closed-loop block diagram*** for each of the following sets of closed-loop equations. Identify all ***signals*** and ***transfer functions***. Then, find the indicated closed-loop transfer function.

1. ***Speed control*** of a car in the presence of a ***disturbance*** force is described by the boxed equations. The ***desired speed*** is $r(t)$, the ***actual speed*** is $v(t)$, the ***speed error*** is $e(t)$, the ***driving force*** on the car is $f_a(t)$, the ***disturbance force*** on the car is $f_d(t)$, and the ***net force*** on the car is $f_{net}(t)$. Find $\frac{V}{R}(s)$.

$$\boxed{\begin{aligned} \frac{dv}{dt} + 3v &= f_{net}(t) \\ f_{net}(t) &= f_a(t) - f_d(t) \\ f_a(t) &= Ke(t) \\ e(t) &= r(t) - v(t) \end{aligned}}$$

2. The boxed equations describe ***position control*** of a spring-mass system using ***proportional*** (P) control. The ***desired position*** of the mass is $r(t)$, the ***actual position*** is $y(t)$, the ***position error*** is $e(t)$, and the ***actuator force*** applied to the mass is $f(t)$. Find $\frac{Y}{R}(s)$.

$$\boxed{\begin{aligned} e(t) &= r(t) - y(t) \\ f(t) &= K\,e(t) \\ \ddot{y} + 4y &= f(t) \end{aligned}}$$

3. The boxed equations describe ***position control*** of a spring-mass-damper system using ***proportional-derivative*** (PD) control. The ***desired position*** of the mass is $r(t)$, the ***actual position*** is $x(t)$, the ***position error*** is $e(t)$, and the ***actuating force*** is $f(t)$. Find $\frac{X}{R}(s)$.

$$\boxed{\begin{aligned} e(t) &= r(t) - x(t) \\ f(t) &= \dot{e}(t) + 5\,e(t) \\ \ddot{x} + 6\dot{x} + 20x &= f(t) \end{aligned}}$$

4. ***Position control*** of a hydraulic actuator using ***proportional-integral*** (PI) control is described by the boxed equations. The desired position is $r(t)$, the actual position is $x(t)$, the position error is $e(t)$, and the actuating forces is $f(t)$. Find the ***transfer function*** $\frac{X}{R}(s)$.

$$\boxed{\begin{aligned} e(t) &= r(t) - x(t) \\ f(t) &= e(t) + 3\int_0^t e(t)dt \\ \ddot{x} + 5\dot{x} &= 2f(t) \end{aligned}}$$

5. The ***speed control*** of a car using proportional-integral (PI) control is described by the boxed equations. The ***desired speed*** is $r(t)$, the ***actual speed*** $v(t)$, the ***speed error*** is $e(t)$, and the ***driving force*** is $f(t)$. Find $\frac{V}{R}(s)$.

$$\boxed{\begin{aligned} m\dot{v} + cv &= f(t) \\ e(t) &= r(t) - v(t) \\ f(t) &= K_1 e(t) + K_2 \int_0^t e(t)dt \end{aligned}}$$

6. The boxed equations describe ***speed control*** of a rotating mass system using proportional control. The ***desired rotational speed*** is $r(t)$, the ***actual rotational speed*** is $\omega(t)$, the ***rotational speed error*** is $e(t)$, the ***input voltage*** to the motor drive actuator is $v(t)$, and the ***actuator torque*** applied to the mass is $M(t)$. Find $\frac{\omega}{R}(s)$.

$$\boxed{\begin{aligned} e(t) &= r(t) - \omega(t) \\ v(t) &= K\,e(t) \\ \dot{M} + 8M &= v(t) \\ \dot{\omega} + 7\omega &= 3M(t) \end{aligned}}$$

7. The boxed equations describe ***position control*** of a spring-mass system using proportional control with a linear actuator. The ***desired position*** of the mass is $r(t)$, the ***actual position*** is $y(t)$, the ***position error*** is $e(t)$, and the ***actuator force*** applied to the mass is $f(t)$. Find $\frac{Y}{R}(s)$.

$$\boxed{\begin{aligned} e(t) &= r(t) - y(t) \\ v(t) &= 10\, e(t) \\ \dot{f} + 3f &= v(t) \\ \ddot{y} + 9y &= f(t) \end{aligned}}$$