

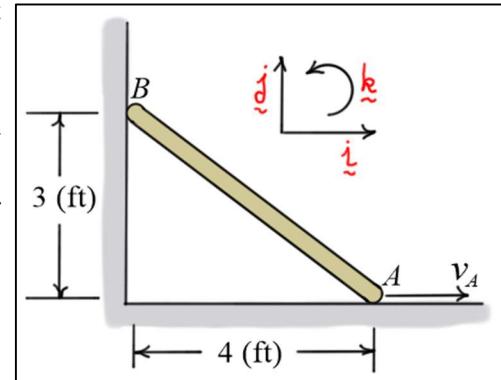
Elementary Dynamics

Exercises #7 – Two-Dimensional Rigid Body Kinematics

1. Bar AB rests against a vertical wall as shown. At the instant shown, the velocity of A is **constant** $v_A = 9\hat{i}$ (ft/s). At this instant, find: a) v_B the velocity of B and ω_{AB} the angular velocity of AB , and b) a_B the acceleration of B and α_{AB} the angular acceleration of AB .

Answers:

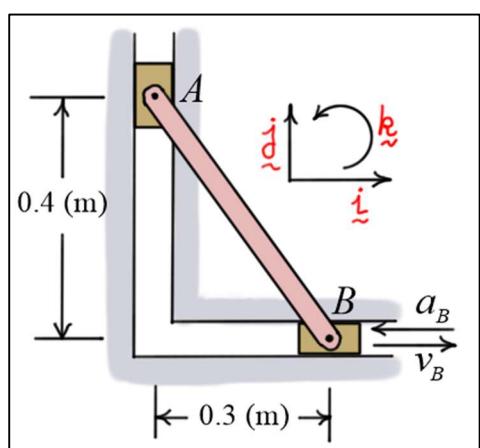
a) $v_B \approx -12\hat{j}$ (ft/s) and $\omega_{AB} \approx 3\hat{k}$ (rad/s)
 b) $a_B \approx -75\hat{j}$ (ft/s²) and $\alpha_{AB} \approx 12\hat{k}$ (rad/s²)



2. Bar AB has its ends constrained to move in the horizontal and vertical slots. At the instant shown, the point B has velocity $v_B = 2\hat{i}$ (m/sec) and acceleration $a_B = -5\hat{i}$ (m/sec²). At this instant, find: a) ω_{AB} the angular velocity of AB and v_A the velocity of A , and b) α_{AB} the angular acceleration of AB and a_A the acceleration of A .

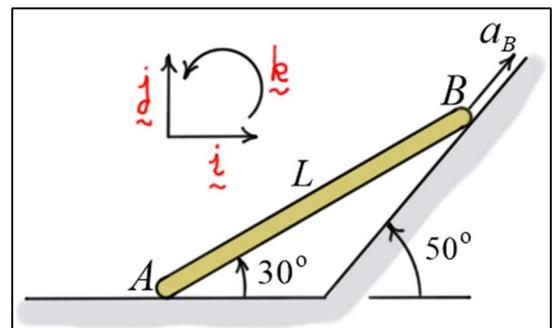
Answers:

a) $\omega_{AB} \approx 5\hat{k}$ (rad/s) and $v_A \approx -1.5\hat{j}$ (m/s);
 b) $\alpha_{AB} \approx 6.25\hat{k}$ (rad/s²) and $a_A \approx -11.9\hat{j}$ (m/s²)

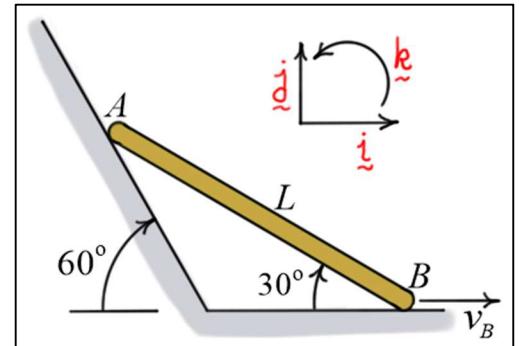


3. The ends of bar AB slide along the horizontal and inclined surfaces. At the instant shown, the angular velocity of AB is $\omega_{AB} = 2\hat{k}$ (rad/s), and the acceleration of end B is $a_B = 1$ (ft/s²) up the inclined plane. At this instant, find α_{AB} the angular acceleration of AB and a_A the acceleration of end A . The length of AB is $L = 5$ (ft).

Answers: $\alpha_{AB} \approx 2.49\hat{k}$ (rad/s²) and $a_A \approx 24.2\hat{i}$ (ft/s²)



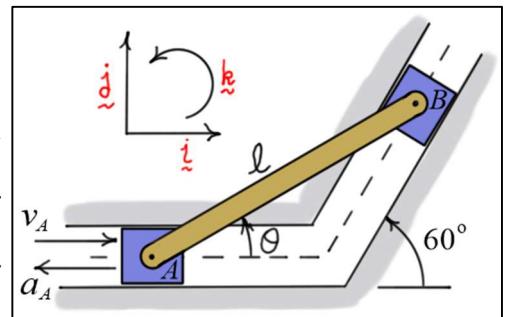
4. The ends of bar AB of length $L = 10$ (in) slide along the horizontal and inclined surfaces as shown. At the instant shown, end B has velocity $\gamma_B = 20 \hat{i}$ (in/sec). a) Using the **relative velocity equation**, find γ_A the velocity of A and ω_{AB} the angular velocity of AB at this instant. b) Repeat part (a) using the concept of **instantaneous centers** of zero velocity.



Answers:

$$|\gamma_A| \approx 20 \text{ (in/s)}; \gamma_A \approx 10 \hat{i} - 17.3 \hat{j} \text{ (in/s)}; \omega_{AB} \approx 2 \hat{k} \text{ (rad/s)}$$

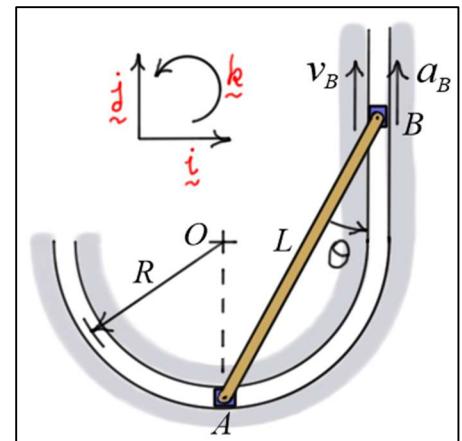
5. The figure shows bar AB of length $\ell = 2$ (ft) whose ends slide along the horizontal and inclined surfaces. At the instant shown, angle $\theta = 30$ (deg), the velocity of A is $\gamma_A = 5 \hat{i}$ (ft/s), and the acceleration of A is $\alpha_A = -10 \hat{i}$ (ft/s²). At this instant, find: a) ω_{AB} the angular velocity of AB and γ_B the velocity of B , and b) α_{AB} the angular acceleration of AB and α_B the acceleration of B .



Answers:

$$\begin{aligned} \text{a) } \omega_{AB} &\approx 2.5 \hat{k} \text{ (rad/s)} \text{ and } \gamma_B \approx 5 \left(\cos(60) \hat{i} + \sin(60) \hat{j} \right) \text{ (ft/s)} \\ \text{b) } \alpha_{AB} &\approx -8.61 \hat{k} \text{ (rad/s}^2\text{)} \text{ and } \alpha_B \approx -24.4 \left(\cos(60) \hat{i} + \sin(60) \hat{j} \right) \text{ (ft/s}^2\text{)} \end{aligned}$$

6. At the instant shown, end A of bar AB moves along a circular slot while end B moves along the straight slot. The radius of the circular slot is $R = 0.2$ (m), the length of the bar is $L = 0.4$ (m), and the angle $\theta = 30$ (deg). At the instant shown, the velocity of B is $\gamma_B = 5 \hat{j}$ (m/s) and the acceleration of B is $\alpha_B = 10 \hat{j}$ (m/s²). a) Using the concept of **instantaneous centers** of zero velocity, find ω_{AB} the angular velocity of AB and γ_A the velocity of A . b) Using the **relative acceleration equation**, find α_{AB} the angular acceleration of AB and α_A the acceleration of A .



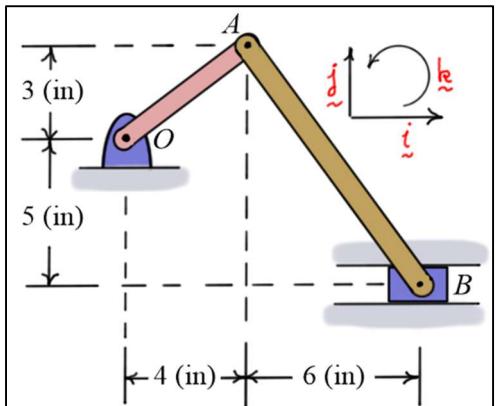
Answers:

$$\begin{aligned} \text{a) } \omega_{AB} &\approx 25 \hat{k} \text{ (rad/s)} \text{ and } \gamma_A \approx 8.66 \hat{i} \text{ (m/s)}; \text{ b) } \alpha_{AB} &\approx -742 \hat{k} \text{ (rad/s}^2\text{)} \text{ and } \alpha_A \approx -132 \hat{i} + 375 \hat{j} \text{ (m/s}^2\text{)} \end{aligned}$$

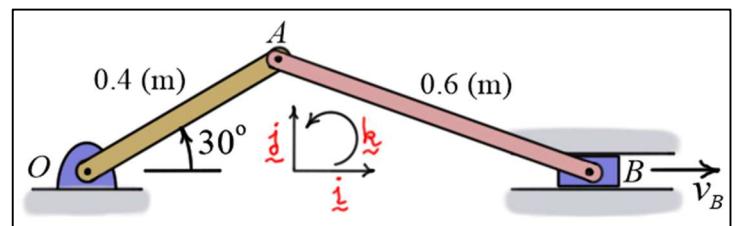
7. The figure shows a slider-crank mechanism OAB . Crank OA is driven at a **constant** angular velocity of $\omega_{OA} = -10\hat{z}$ (rad/sec). a) Using the **relative velocity equation**, find ω_{AB} the angular velocity of the connecting bar AB and v_B the velocity of the slider B . b) Repeat part (a) using the concept of **instantaneous centers** of zero velocity.

Answers:

$$\omega_{AB} \approx 6.67\hat{z} \text{ (rad/s)} \text{ and } v_B \approx 83.3\hat{z} \text{ (in/s)}$$



8. The figure shows slider-crank mechanism OAB . At the instant shown, the velocity of B is $v_B = 3\hat{z}$ (m/s). Using the concept of **instantaneous centers** of zero velocity, find ω_{AB} the angular velocity of the connecting rod AB and v_A the velocity of A at this instant.

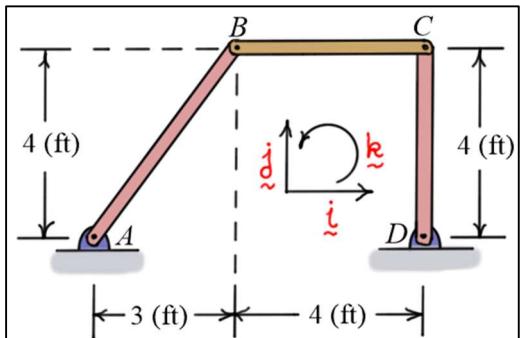


Answers: $\omega_{AB} \approx 5.7\hat{z}$ (rad/s) and $v_A \approx 3.72(\sin(30)\hat{z} - \cos(30)\hat{j})$

9. At the instant shown, the **angular velocity** of bar AB of the four-bar mechanism $ABCD$ is $\omega_{AB} = 10\hat{z}$ (rad/s). Using the concept of **instantaneous centers** of zero velocity, find ω_{BC} and ω_{CD} the angular velocities of bars BC and CD .

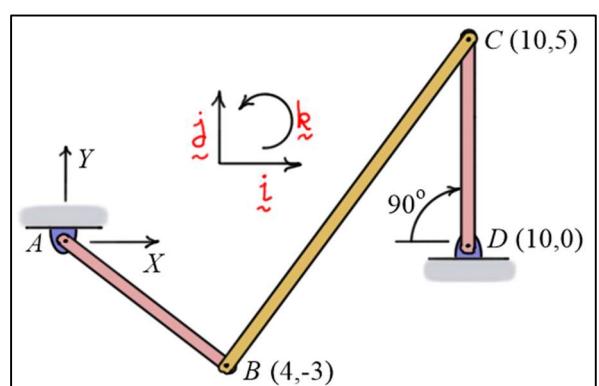
Answers:

$$\omega_{BC} \approx -7.5\hat{z} \text{ (rad/s)}; \omega_{CD} \approx 10\hat{z} \text{ (rad/s)}$$

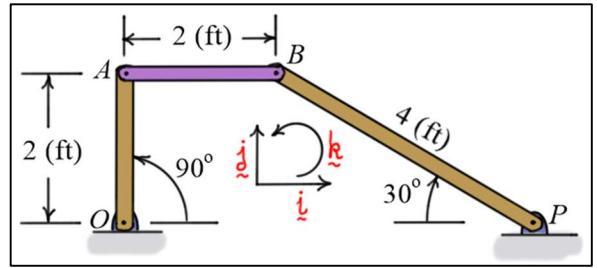


10. The figure shows a four-bar mechanism $ABCD$. Point A is located at the origin, and points B , C and D have the coordinates shown. At the instant shown, link AB has a **constant** angular velocity of $\omega_{AB} = 9\hat{z}$ (rad/s). a) Using the **relative velocity equation**, find ω_{BC} and ω_{CD} the angular velocities of links BC and CD . b) Repeat part (a) using the concept of **instantaneous centers** of zero velocity.

Answers: $\omega_{BC} \approx -6\hat{z}$ (rad/s) and $\omega_{CD} \approx -15\hat{z}$ (rad/s)



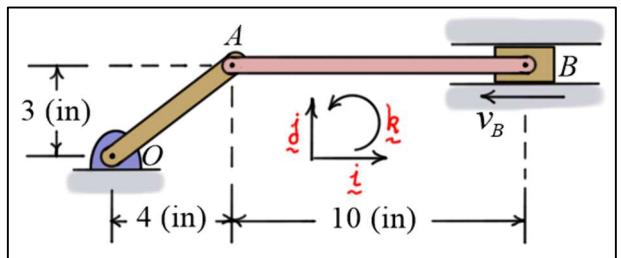
11. The figure shows a four-bar mechanism $OABP$. At the instant shown, the angular velocity of the crank OA is $\omega_{OA} = 5\hat{k}$ (rad/s). Using the concept of **instantaneous centers** of zero velocity, find ω_{AB} the angular velocity of connecting bar AB .



Answer: $\omega_{AB} \approx -8.66\hat{k}$ (rad/s)

12. The figure shows a slider-crank mechanism OAB . At the instant shown, the velocity of slider B is $v_B = -6\hat{i}$ (in/s).

a) Using the **relative velocity equation**, find ω_{AB} the angular velocity of connecting rod AB and ω_{OA} the angular velocity of crank OA . b) Repeat part (a) using the concept of **instantaneous centers** of zero velocity.

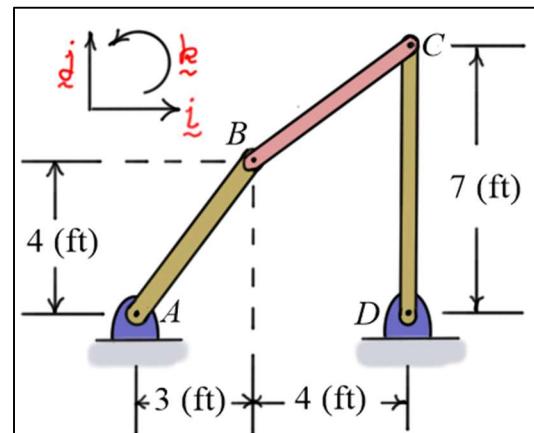


Answers: $\omega_{AB} \approx -0.8\hat{k}$ (rad/s) and $\omega_{OA} \approx 2\hat{k}$ (rad/s)

13. At the instant shown, the angular velocity of bar AB of the four-bar mechanism $ABCD$ is $\omega_{AB} = 10\hat{k}$ (rad/s). Using the **relative velocity equation**, find a) v_B the velocity of point B , and b) ω_{BC} and ω_{CD} the angular velocities of bars BC and CD .

Answers:

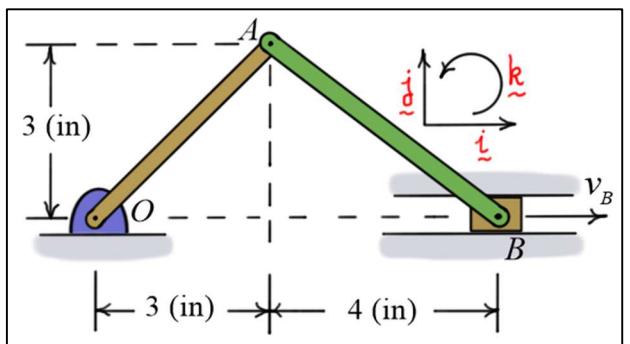
a) $v_B \approx -40\hat{i} + 30\hat{j}$ (ft/s)
b) $\omega_{BC} \approx -7.5\hat{k}$ (rad/s); $\omega_{CD} \approx 2.5\hat{k}$ (rad/s)



14. The figure shows a slider-crank mechanism OAB . The slider B moves at a **constant velocity** $v_B = 10.5\hat{i}$ (in/s). At the instant shown, find: a) ω_{OA} and ω_{AB} the angular velocities of the two bars, and b) α_{OA} and α_{AB} the angular accelerations of the bars.

Answers:

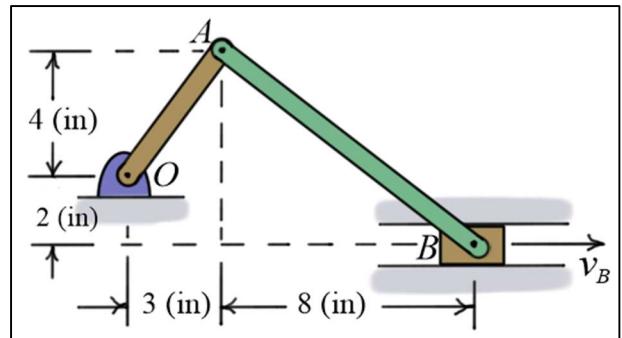
a) $\omega_{OA} = -2\hat{k}$ (rad/s), $\omega_{AB} = +1.5\hat{k}$ (rad/s); b) $\alpha_{OA} \approx -3.25\hat{k}$ (rad/s²), $\alpha_{AB} \approx 3.75\hat{k}$ (rad/s²)



15. The figure shows a slider-crank mechanism OAB . The slider has **constant** velocity $v_B = 20\hat{i}$ (in/sec). At the instant shown, find: a) ω_{OA} and ω_{AB} the angular velocities of OA and AB , and b) α_{OA} and α_{AB} the angular accelerations of OA and AB .

Answers:

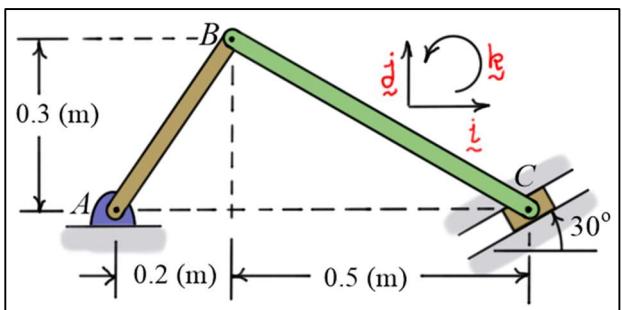
a) $\omega_{OA} = -3.2\hat{k}$ (rad/sec) and $\omega_{AB} = 1.2\hat{k}$ (rad/sec);
b) $\alpha_{OA} \approx -2.88\hat{k}$ (rad/s²) and $\alpha_{AB} \approx 5.12\hat{k}$ (rad/s²)



16. The figure shows a slider-crank mechanism ABC . Bar AB has a **constant** angular velocity of $\omega_{AB} = -10\hat{k}$ (rad/sec). Using the equations for relative velocity and acceleration, find: a) ω_{BC} the angular velocity of BC and v_C the velocity of C , and b) α_{BC} the angular acceleration of BC and α_C the acceleration of C .

Answers:

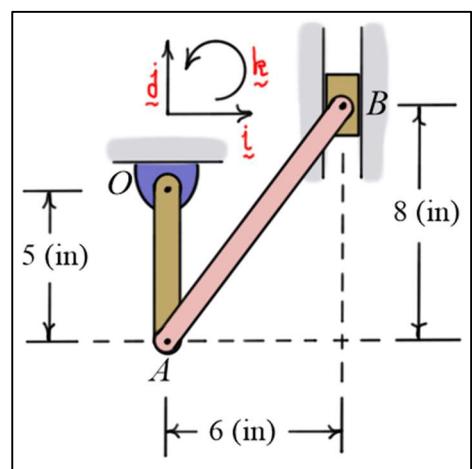
a) $\omega_{BC} \approx 11.4\hat{k}$ (rad/s) and $v_C \approx 7.42(\cos(30)\hat{i} + \sin(30)\hat{j})$ (m/s)
b) $\alpha_{BC} \approx -178\hat{k}$ (rad/s²) and $\alpha_C \approx -160(\cos(30)\hat{i} + \sin(30)\hat{j})$ (m/s²)



17. The figure shows a slider-crank mechanism OAB . Crank OA is driven at a **constant** rate of $\omega_{OA} = 8\hat{k}$ (rad/sec). At the instant shown, find: a) ω_{AB} the angular velocity of AB and v_B the velocity of B , and b) α_{AB} the angular acceleration of AB and α_B the acceleration of B .

Answers:

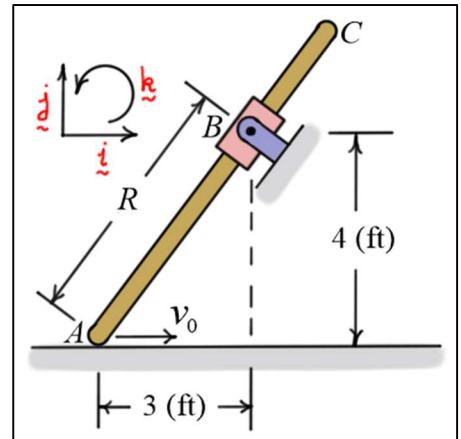
a) $\omega_{AB} \approx 5\hat{k}$ (rad/sec) and $v_B \approx 30\hat{j}$ (in/s)
b) $\alpha_{AB} \approx -18.8\hat{k}$ (rad/s²) and $\alpha_B \approx 7.5\hat{j}$ (in/s²)



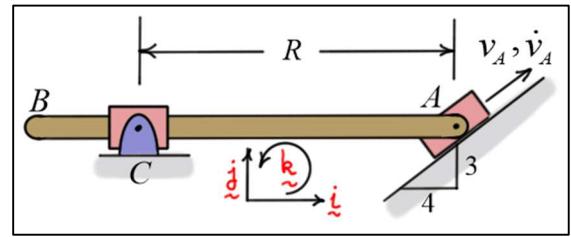
18. Bar AC rests on the horizontal plane at A and slides through a **smooth collar** at B . The collar rotates freely to allow the bar to rotate as A moves at a **constant** speed of $v_0 = 10 \hat{j}$ (ft/s). The variable distance from A to B is R . At the instant shown, find: a) ω_{AC} the angular velocity of AC and \dot{R} the time rate of change of the distance R , and b) α_{AC} the angular acceleration of AC and \ddot{R} the time rate of change of \dot{R} .

Answers:

a) $\omega_{AC} \approx 1.6 \hat{k}$ (rad/s); $\dot{R} \approx -6.0$ (ft/s)
b) $\alpha_{AC} \approx 3.84 \hat{k}$ (rad/s²); $\ddot{R} \approx 12.8$ (ft/s²)



19. Bar AB slides through the collar at C while its end A moves up the inclined plane. The collar rotates freely to allow the bar to rotate as A moves up the plane. At the instant shown, the velocity and acceleration of A are $v_A = 10$ (ft/s) and $\dot{v}_A = 5$ (ft/s²), and the variable distance from C to A is $R = 2$ (ft).



At this instant, find: a) ω_{AB} the angular velocity of AB and \dot{R} the time rate of change of R , and b) α_{AB} the angular acceleration of AB and \ddot{R} the time rate of change of \dot{R} .

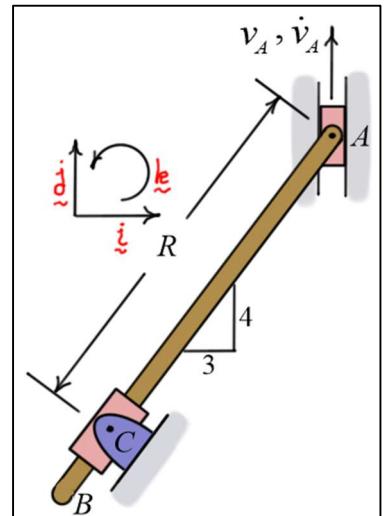
Answers:

a) $\omega_{AB} \approx 3 \hat{k}$ (rad/s) and $\dot{R} \approx 8$ (ft/s); b) $\alpha_{AB} \approx -22.5 \hat{k}$ (rad/s²) and $\ddot{R} \approx 22$ (ft/s²)

20. Bar AB slides through a collar at C while its end A moves in the vertical slot. The collar rotates freely to allow the bar to rotate as A moves up. At the instant shown, the velocity and acceleration of A are $v_A = 12.5 \hat{j}$ (ft/s) and $\dot{v}_A = 5 \hat{j}$ (ft/s²), and the variable distance from C to A is $R = 5$ (ft). At this instant, find a) ω_{AB} the angular velocity of AB and \dot{R} the time rate of change of R , and b) α_{AB} the angular acceleration of AB and \ddot{R} the time rate of change of \dot{R} .

Answers: a) $\omega_{AB} = 1.5 \hat{k}$ (rad/s), $\dot{R} = 10$ (ft/s);

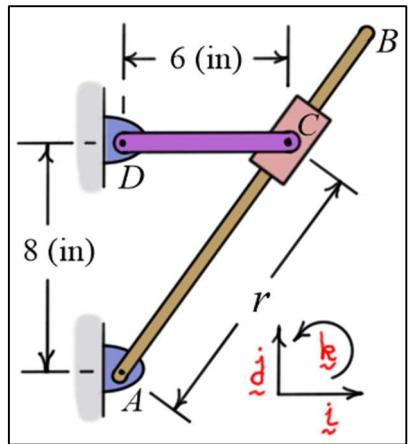
b) $\alpha_{AB} \approx -5.4 \hat{k}$ (rad/s²) and $\ddot{R} \approx 15.3$ (ft/s²)



21. The system shown consists of two bars AB and CD connected by a collar at C . Bar AB is free to slide through the collar as it rotates at a **constant rate** of $\omega_{AB} = 18 \text{ rad/s}$, and bar CD is pinned to the collar at C . The variable distance from A to C is represented by the symbol r . At the instant shown, find: a) ω_{CD} the angular velocity of CD and \dot{r} the time rate of change of the distance r , and b) α_{CD} the angular acceleration of CD and \ddot{r} the time rate of change of \dot{r} .

Answers:

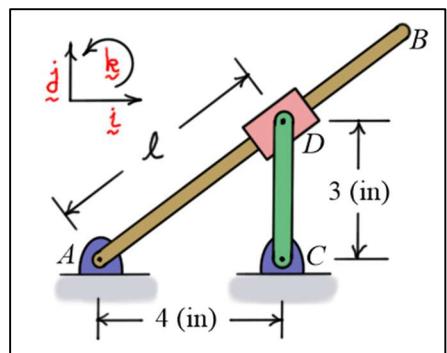
a) $\omega_{CD} \approx 50 \text{ rad/s}$ and $\dot{r} \approx 20 \text{ ft/s}$; b) $\alpha_{CD} \approx -933 \text{ rad/s}^2$ and $\ddot{r} \approx -853 \text{ ft/s}^2$



22. The system shown consists of two bars AB and CD and a collar at D . The collar is pinned to bar CD and is free to slide along and rotate with bar AB . The variable distance between A and D is ℓ . Bar CD rotates at a **constant rate** $\omega_{CD} = 10 \text{ rad/s}$. At the instant shown, find: a) ω_{AB} the angular velocity of AB and $\dot{\ell}$ the time rate of change of ℓ , and b) α_{AB} the **angular acceleration** of AB , and $\ddot{\ell}$ the time derivative of $\dot{\ell}$.

Answers:

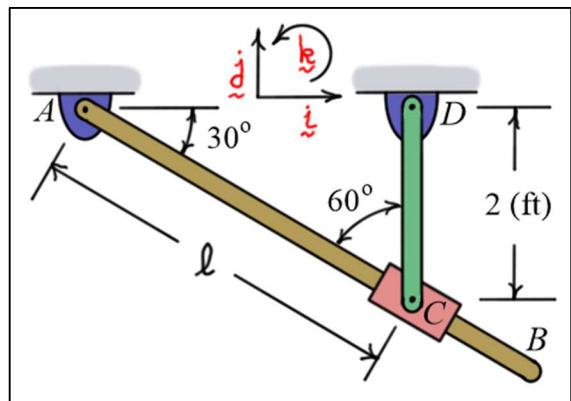
a) $\omega_{AB} = 3.6 \text{ rad/s}$ and $\dot{\ell} = -24 \text{ in/s}$; b) $\alpha_{AB} \approx -13.4 \text{ rad/s}^2$ and $\ddot{\ell} \approx -115 \text{ in/s}^2$



23. The system shown consists of two bars AB and CD and a collar at C . The collar is pinned to bar CD and is free to slide along and rotate with bar AB . The **variable length** between A and C is ℓ . Bar CD rotates at a **constant rate** of $\omega_{CD} = 5 \text{ rad/s}$. At the instant shown when $\ell = 4 \text{ ft}$, find: a) ω_{AB} the angular velocity of AB and $\dot{\ell}$ the time derivative of ℓ , and b) α_{AB} the angular acceleration of AB and $\ddot{\ell}$ the time derivative of $\dot{\ell}$.

Answers:

a) $\omega_{AB} \approx 1.25 \text{ rad/s}$ and $\dot{\ell} \approx 8.66 \text{ ft/s}$; b) $\alpha_{AB} \approx 5.41 \text{ rad/s}^2$ and $\ddot{\ell} \approx -18.8 \text{ ft/s}^2$



24. The system shown consists of two bars AB and CD connected by a collar at C . Bar AB is free to slide through the collar as it rotates, and bar CD is pinned to the collar. Length ℓ represents the **variable distance** from A to C . At the instant shown, length $\ell = 4$ (in) and the angular velocity of AB is **constant** $\omega_{AB} = 5 \hat{k}$ (rad/s). At this instant, find: a) ω_{CD} the angular velocity of CD and $\dot{\ell}$ the first derivative of the distance ℓ , and b) α_{CD} the angular acceleration of CD and $\ddot{\ell}$ the time derivative of $\dot{\ell}$.

Answers:

a) $\omega_{CD} \approx 5 \hat{k}$ (rad/s) and $\dot{\ell} \approx 15$ (in/s); b) $\alpha_{CD} \approx 18.8 \hat{k}$ (rad/s²) and $\ddot{\ell} \approx 56.3$ (in/s²)

25. The system shown consists of two bars AB and CD connected by a collar at B . Bar CD is free to slide through the collar as it rotates, and bar AB is pinned to the collar. The length ℓ represents the **variable distance** from C to B . At the instant shown, length $\ell = 0.5$ (m) and the angular velocity of AB is **constant** $\omega_{AB} = 10 \hat{k}$ (rad/s). At this instant, find: a) ω_{CD} the angular velocity of CD and $\dot{\ell}$ the time derivative of the length ℓ , and b) α_{CD} the angular acceleration of CD and $\ddot{\ell}$ the time derivative of $\dot{\ell}$.

Answers:

a) $\omega_{CD} \approx 6.4 \hat{k}$ (rad/s) and $\dot{\ell} \approx 2.4$ (m/s); $\alpha_{CD} \approx -13.4 \hat{k}$ (rad/s²) and $\ddot{\ell} \approx -11.5$ (m/s²)

