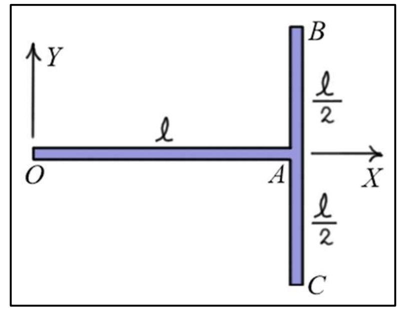
Elementary Dynamics
Exercises #8 – Two-Dimensional Rigid Body Kinetics: Inertia and Newton's Laws

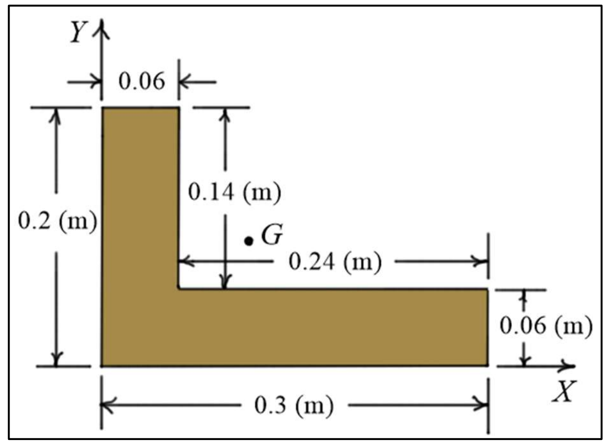
1. The figure shows a T-shaped bracket consisting to two long slender bars each having mass m and length ℓ. Find I_O the moment of inertia of the bracket about the *Z*-axis passing through point *O*.

   Answer:
   $I_O = \frac{17}{12} m\,\ell^2$



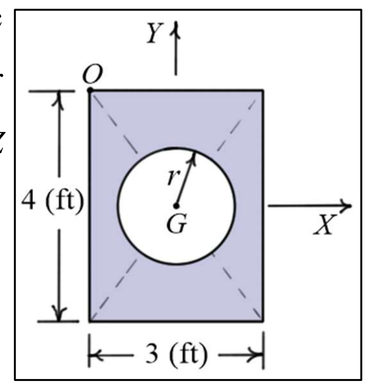
2. The figure shows a thin L-shaped plate of mass $m = 5$ (kg). All dimensions of the plate are given in meters. a) Find *G* the mass-center of the plate relative to the *XY* axes shown. b) Find I_G the moment of inertia of the plate about the *Z* axis passing through *G*.

   Answers:
   a) $\underset{\sim}{r}_G \approx 0.11185\,\underset{\sim}{i} + 0.06183\,\underset{\sim}{j}$ (m);
   b) $I_G \approx 0.05615$ (kg-m^2)



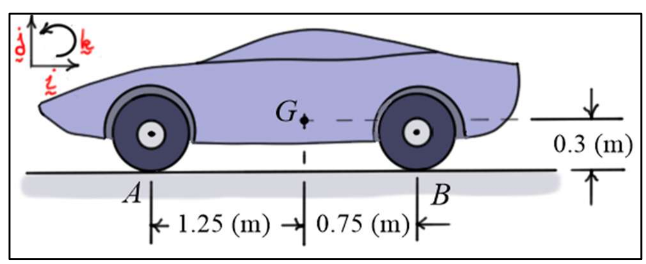
3. The figure shows a thin rectangular plate with a circular hole at its geometric center. The plate has weight per unit area $W/A = 10$ (lb/ft^2), and the circular hole has radius $r = 1$ (ft). Find I_O the moment of inertia of the plate about the *Z* axis passing through the corner point *O*.

   Answer: $I_O \approx 24.5$ (slug-ft^2)

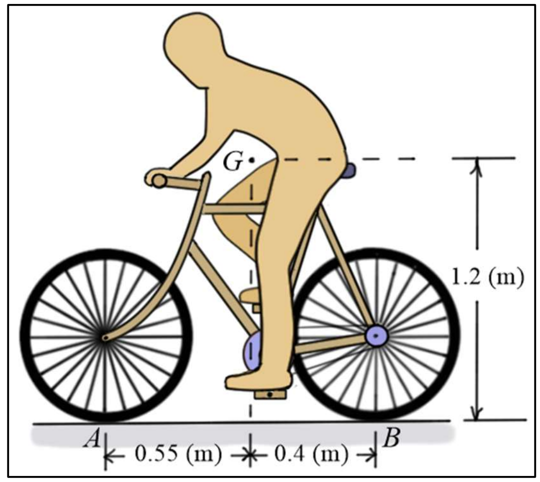


4. A car is traveling forward when its breaks are suddenly applied so all wheels stop rotating and slide on the surface. The mass of the car is $m = 2000$ (kg) and the kinetic coefficient of friction between the tires and the surface is $\mu_k = 0.25$. Find: a) $\underset{\sim}{a}_G$ the acceleration of the car, and b) $\underset{\sim}{N}_A$ and $\underset{\sim}{N}_B$ the normal forces acting on the tires at *A* and *B* as the car skids to a stop.

   Answers:
   a) $\underset{\sim}{a}_G \approx 2.45\,\underset{\sim}{i}$ (m/s^2); b) $\underset{\sim}{N}_A \approx 8.09\,\underset{\sim}{j}$ (kN) and $\underset{\sim}{N}_B \approx 11.5\,\underset{\sim}{j}$ (kN)



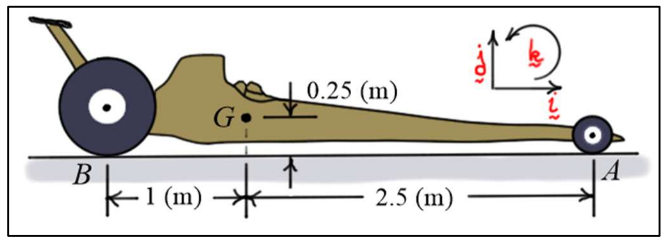
5. The bicycle and rider have a combined mass of 80 (kg) with center of mass G as shown. Find $(\mu_k)_{\min}$ the minimum coefficient of kinetic friction between the road and the wheels so the rear wheel just starts to lift off the ground at B when the rider applies the brakes to the front wheel. Neglect the mass and inertia of the wheels and assume the front wheel slides on the surface.



Answer:

$(\mu_k)_{\min} \approx 0.458$

6. The dragster has mass $m = 1500$ (kg) and center of mass at G as shown. Neglect the size and mass of the wheels, and assume the front wheels are free to roll. a) Find $\underset{\sim}{f}_{\text{total}}$ the total friction force on the rear tires necessary to cause the total normal force on the front wheels to be zero.



b) If the coefficient of static friction between the rear tires and the road is $\mu_S = 0.6$, is it possible to lift the front tires off the road. Why?

Answers:

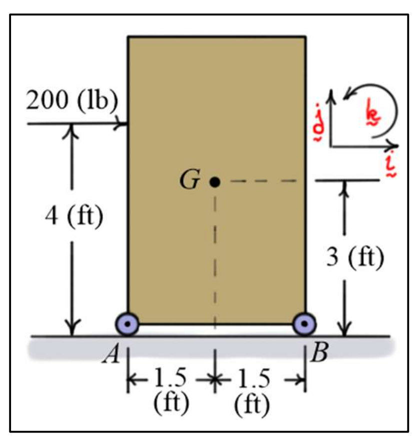
a) $\underset{\sim}{f}_{\text{total}} \approx 58.9\,\underset{\sim}{i}$ (kN); b) $\left(\underset{\sim}{f}_{\text{total}}\right)_{\max} \approx 8.83\,\underset{\sim}{i}$ (kN); It is not possible. There is not enough friction.

7. Assuming the static coefficient of friction for the rear wheels of the dragster shown in problem 6 is $\mu_S = 0.6$ and that the rear wheels ***do not slip***, find: a) $\underset{\sim}{a}_{\max}$ the maximum acceleration of the dragster, and b) $\underset{\sim}{N}_A$ and $\underset{\sim}{N}_B$ the corresponding normal forces on the front and rear tires. Neglect the size and mass of the wheels, and assume the front wheels are free to roll.

Answers:

a) $\underset{\sim}{a}_{\max} \approx 4.39\,\underset{\sim}{i}$ (m/s^2); b) $\underset{\sim}{N}_A \approx 3.73\,\underset{\sim}{j}$ (kN), $\underset{\sim}{N}_B \approx 11.0\,\underset{\sim}{j}$ (kN)

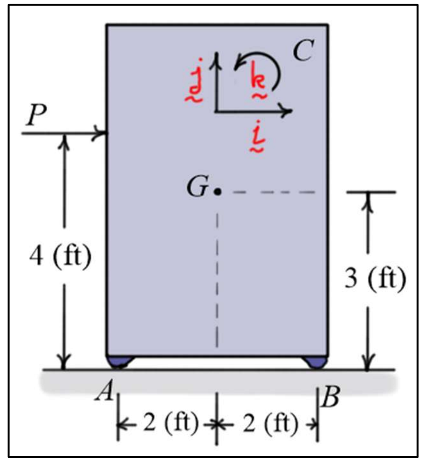
8. The figure shows a 500 (lb) crate on rollers that is moved along the floor by the 200 (lb) force as shown. Given that when the force is applied, the crate translates to the right with no rotation, find a) $\underset{\sim}{a}_G$ the acceleration of the mass-center G, and b) $\underset{\sim}{N}_A$ and $\underset{\sim}{N}_B$ the normal forces the ground exerts on the rollers at A and B.



Answers:

a) $\underset{\sim}{a}_G \approx 12.9\,\underset{\sim}{i}$ (ft/s^2); b) $\underset{\sim}{N}_A \approx 183\,\underset{\sim}{j}$ (lb) and $\underset{\sim}{N}_B \approx 317\,\underset{\sim}{j}$ (lb)

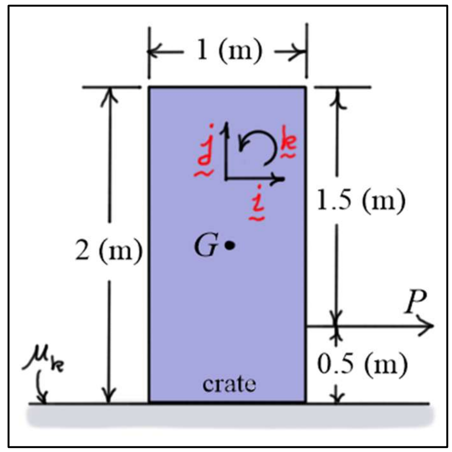
9. The crate C of weight $W = 500$ (lb) rests on a horizontal plane (floor) as shown. The horizontal force $P = 200$ (lb), located 4 (ft) above the floor, causes the crate to slide to the right. The coefficient of kinetic friction between the skids (at A and B) and the floor is $\mu_K = 0.2$. Find: a) $\underset{\sim}{a}_G$ the acceleration of the mass-center G, and b) $\underset{\sim}{N}_A$ and $\underset{\sim}{N}_B$ the normal forces applied to the crate by the floor at the skids at A and B.



Answers:

a) $\underset{\sim}{a}_G \approx 6.44\,\underset{\sim}{i}$ (ft/s^2); b) $\underset{\sim}{N}_A \approx 125\,\underset{\sim}{j}$ (lb) and $\underset{\sim}{N}_B \approx 375\,\underset{\sim}{j}$ (lb)

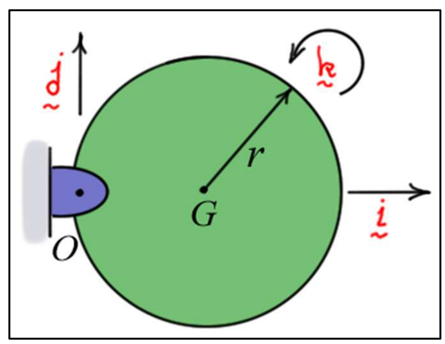
10. The figure shows a crate resting on a horizontal plane (floor). It is pulled to the right by a force $P = 450$ (N) located 0.5 (m) above the floor. The crate has mass $m = 50$ (kg), and its mass-center G is in the geometric center of the crate. The coefficient of kinetic friction between the crate and the floor is $\mu_k = 0.2$. a) Find the location of the resultant normal force acting on the bottom of the crate and verify the crate will not tip. b) Find $\underset{\sim}{a}_G$ the acceleration of the mass center G.



Answers:

a) Resultant normal force is located $x \approx 0.26$ (m) left of centerline. Crate doesn't tip because $x < 0.5$ (m).
b) $\underset{\sim}{a}_G \approx 7.04\,\underset{\sim}{i}$ (m/s^2)

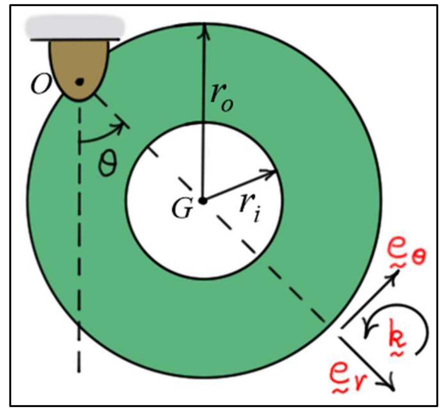
11. The figure shows a thin disk that is pinned and rotates freely about the fixed-point O. The disk has radius $r = 0.5$ (ft) and weight $W = 15$ (lb). At the instant shown, the angular velocity of the disk is $\underset{\sim}{\omega}_D = -9.5\,\underset{\sim}{k}$ (rad/s). At this instant, find a) $\underset{\sim}{\alpha}_D$ the angular acceleration of the disk, and b) $\underset{\sim}{F}_O$ the force the support applies to the disk at O.



Answers:

a) $\underset{\sim}{\alpha}_D \approx -42.9\,\underset{\sim}{k}$ (rad/s^2); b) $\underset{\sim}{F}_O \approx 21\,\underset{\sim}{i} + 5\,\underset{\sim}{j}$ (lb)

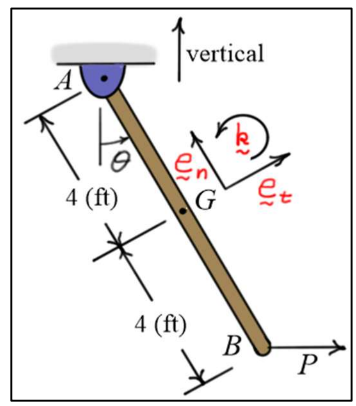
12. The thin annular plate P has outer radius $r_o = 1.0$ (m) and inner radius $r_i = 0.5$ (m). The plate has mass density $\rho = 10$ (kg/m^2). The plate is released from rest at $\theta = 60$ (deg). a) Find I_O the moment of inertia of the plate about the pivot point O. b) Find $\underset{\sim}{\alpha}_P$ the angular acceleration of the plate and $\underset{\sim}{F}_O$ the force applied to the plate by the support at O at the instant of release.



Answers:

a) $I_O \approx 38.3$ (kg-m^2); b) $\underset{\sim}{\alpha}_P \approx -5.23\underset{\sim}{k}$ (rad/s^2) and $\underset{\sim}{F}_O \approx -116\underset{\sim}{e}_r + 77\underset{\sim}{e}_\theta$ (N)

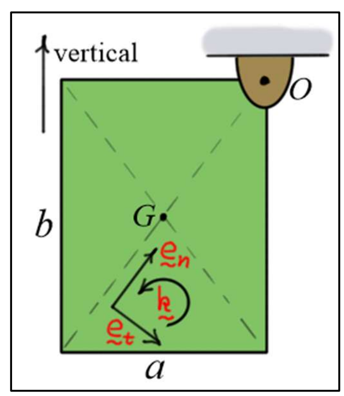
13. The long slender bar AB is pinned to a fixed support at A and is pulled to the right by the force $P = 50$ (lb). The bar has length $L = 8$ (ft) and weight $W = 30$ (lb). The bar is released from rest at $\theta = 30$ (deg). At this instant, find: a) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of AB, and b) $\underset{\sim}{F}_A$ the force the support at A applies to the bar.



Answers:

a) $\underset{\sim}{\alpha}_{AB} \approx 14.4\underset{\sim}{k}$ (rad/s^2); b) $\underset{\sim}{F}_A \approx 51.0\underset{\sim}{e}_n + 25.4\underset{\sim}{e}_t$ (lb)

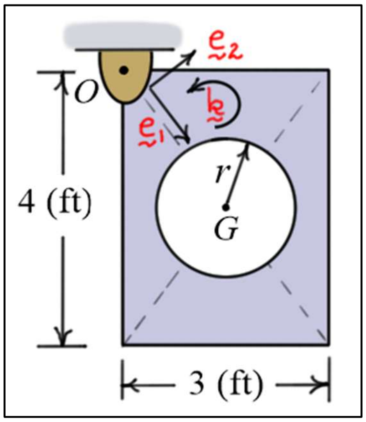
14. The thin rectangular plate is pinned so it pivots about its upper right corner as shown. The plate has weight $W = 15$ (lb), and it has sides have lengths of $a = 0.75$ (ft) and $b = 1$ (ft). At the instant shown, the plate has an angular velocity of $\underset{\sim}{\omega} = 10\underset{\sim}{k}$ (rad/s). At this instant, find a) $\underset{\sim}{\alpha}$ the angular acceleration of the plate, and b) $\underset{\sim}{F}_O$ the force applied to the plate by the support at O. The moment of inertia of the thin plate about its mass-center is $I_G = \frac{1}{12}m(a^2 + b^2)$.



Answers:

a) $\underset{\sim}{\alpha} \approx 23.2\underset{\sim}{k}$ (rad/s^2); b) $\underset{\sim}{F}_O \approx 41.1\underset{\sim}{e}_n - 2.25\underset{\sim}{e}_t$

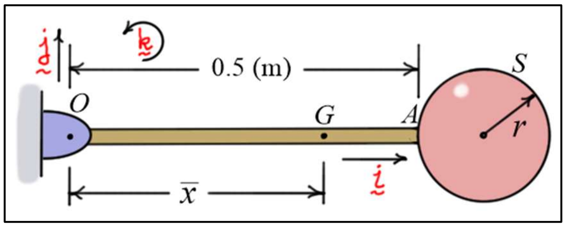
15. The system shown consists of a thin rectangular plate with a circular hole at its geometric center. The plate has weight per unit area $W/A = 10$ (lb/ft^2), and the circular hole has radius $r = 1$ (ft). The plate is pinned and rotates about its upper left corner at O. Assuming the plate is released from rest in the position shown, find a) $\underset{\sim}{\alpha}$ the angular acceleration of the plate, and b) $\underset{\sim}{F}_O$ the force the support at O exerts on the plate.



Answers:

a) $\underset{\sim}{\alpha} \approx -5.43\underset{\sim}{k}$ (rad/s^2); b) $\underset{\sim}{F}_O \approx -70.9\underset{\sim}{e}_1 + 15.8\underset{\sim}{e}_2$

16. The compound pendulum shown below consists of a 20 (kg) slender bar OA and a 10 (kg) sphere S of radius $r = 0.1$ (m) as shown. In the position shown, the angular velocity of the pendulum is $\underset{\sim}{\omega} = -2\underset{\sim}{k}$ (rad/s). a) Find $\bar{x}$ the distance from O to G the mass-center of the pendulum. b) Find I_O the moment of inertia of the pendulum about the pivot point O. c) In this position, find $\underset{\sim}{\alpha}$ the angular acceleration of the pendulum and $\underset{\sim}{F}_O$ the force the support applies to the pendulum at O.

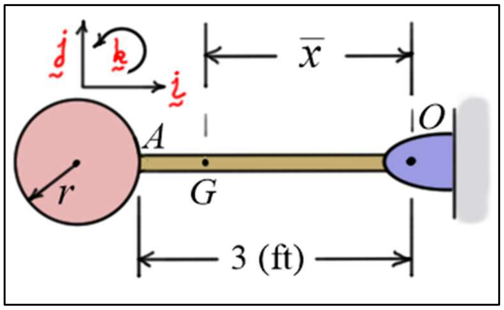


Answers:

a) $\bar{x} \approx 0.366$ (m); b) $I_O \approx 5.31$ (kg-m^2)

c) $\underset{\sim}{\alpha} \approx -20.3\underset{\sim}{k}$ (rad/s^2), $\underset{\sim}{F}_O \approx -44\underset{\sim}{i} + 70.6\underset{\sim}{j}$ (N)

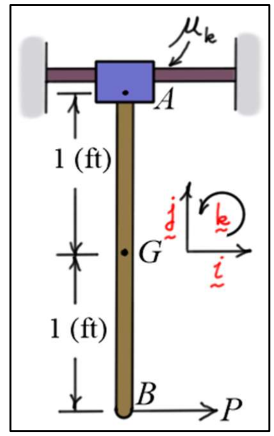
17. The compound pendulum shown consists of a 10 (lb) slender bar OA of length $\ell = 3$ (ft) and a 15 (lb) disk of radius $r = 0.75$ (ft). In the position shown, its angular velocity is $\underset{\sim}{\omega} = 8\underset{\sim}{k}$ (rad/s). a) Find $\bar{x}$ the distance from O to G the mass center of the pendulum. b) Find I_O the moment of inertia of the pendulum about the pivot point O. c) At the instant shown, find $\underset{\sim}{F}_O$ the force the support exerts on the pendulum at O and $\underset{\sim}{\alpha}$ the angular acceleration of the pendulum.



Answers:

a) $\bar{x} \approx 2.85$ (ft); b) $I_O \approx 7.61$ (slug-ft^2); c) $\underset{\sim}{F}_O \approx 142\underset{\sim}{i} + 4.29\underset{\sim}{j}$ (lb) and $\underset{\sim}{\alpha} \approx 9.36\underset{\sim}{k}$ (rad/s^2)

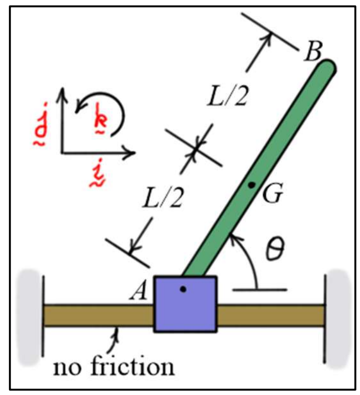
18. The system shown consists of a slender bar AB pinned to a light (massless) collar at A. The bar has weight $W = 4$ (lb) and length $L = 2$ (ft). The collar slides along the horizontal bar with kinetic coefficient of friction $\mu_k = 0.2$. The bar is initially at rest when the force $P = 10$ (lb) is applied to end B. At this instant, find a) $\underset{\sim}{a}_G$ the acceleration of the mass center G, and b) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of AB.



Answers:

a) $\underset{\sim}{a}_G \approx 74.1\underset{\sim}{i}$ (ft/s^2); b) $\underset{\sim}{\alpha}_{AB} \approx 261\underset{\sim}{k}$ (rad/s^2)

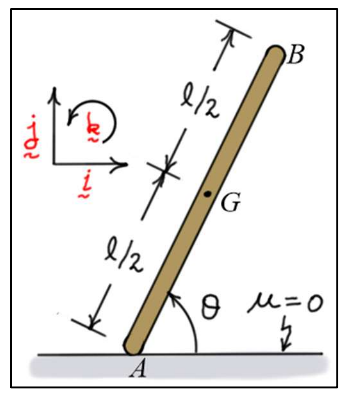
19. The figure shows a slender bar AB of length of $L = 4$ (ft) and weight $W = 19$ (lb). Its lower end at A is pinned to a light (massless) collar that moves only in the $\underset{\sim}{i}$ direction. Given AB is released from ***rest*** when $\theta = 60^\circ$, find: a) $\underset{\sim}{F}_A$ the force transmitted through the pin at A, and b) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of AB.



Answers:

a) $\underset{\sim}{F}_A \approx 10.9\,\underset{\sim}{j}$ (lb); b) $\underset{\sim}{\alpha}_{AB} \approx -13.8\,\underset{\sim}{k}$ (rad/s^2)

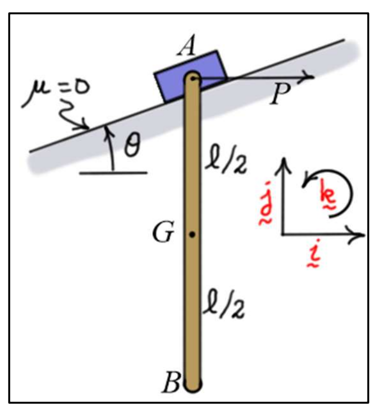
20. The slender bar AB is released from rest at an angle $\theta = 60$ (deg) with end A touching the smooth (frictionless) horizontal plane. The bar has weight $W = 10$ (lb) and length $\ell = 2$ (ft). At the instant the bar is released, find: a) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of AB, b) $\underset{\sim}{N}$ the normal force the surface exerts on AB, and c) $\underset{\sim}{a}_A$ the acceleration of end A.



Answers:

a) $\underset{\sim}{\alpha}_{AB} \approx -27.6\,\underset{\sim}{k}$ (rad/s^2); b) $\underset{\sim}{N} \approx 5.71\,\underset{\sim}{j}$ (lb); c) $\underset{\sim}{a}_A \approx -23.9\,\underset{\sim}{i}$ (ft/s^2)

21. The slender bar AB of weight $W = 10$ (lb) and length $\ell = 2$ (ft) is hanging at rest when the force $P = 50$ (lb) is applied causing the block to move up the smooth (frictionless) plane and the bar to swing. The angle of the plane is $\theta = 30$ (deg). At the instant P is applied find: a) a_A the acceleration of A, b) $\underset{\sim}{\alpha}_{AB}$ the angular acceleration of AB, and c) N the normal force the plane exerts on the block. Neglect the mass of the block.



Answers:

a) $a_A \approx 282$ (ft/s^2) (up the plane); b) $\underset{\sim}{\alpha}_{AB} \approx -183\underset{\sim}{k}$ (rad/s^2); c) $N \approx 62.1$ (lb)