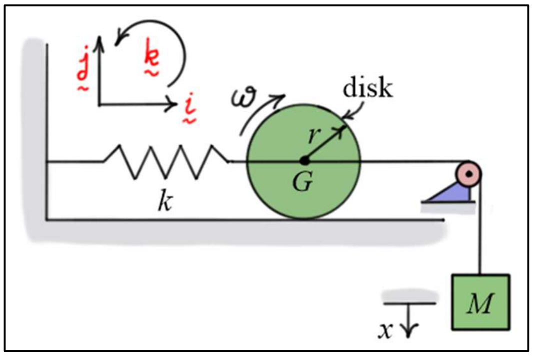
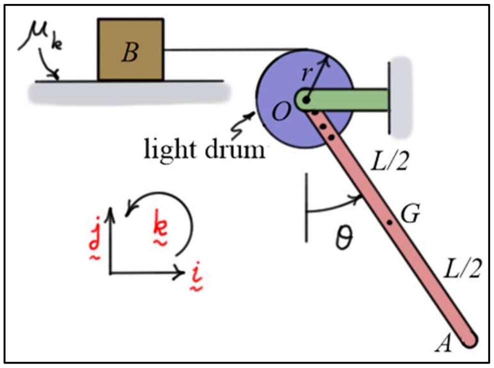
Elementary Dynamics
Exercises #9 – Two-Dimensional Rigid Body Kinetics: Work and Energy

1. The figure shows a system consisting of a $2\ \text{(kg)}$ block M and a $5\ \text{(kg)}$ disk of radius $r = 0.1$ (m). As the block moves down the disk rolls without slipping on the horizontal plane. The spring has stiffness $k = 56\ \text{(N/m)}$. Given that the system is released from rest with the spring unstretched, find $\underset{\sim}{\omega}$ the angular velocity of the disk after the block moves down 0.5 meters.



Answer:
$\underset{\sim}{\omega} \approx -7.69\,\underset{\sim}{k}\ \text{(rad/s)}$

2. The system shown consists of bar OA, a light drum, block B, and a connecting cable. The bar and drum are riveted together so they rotate as a single body. As the bar and drum rotate about O, block B is pulled along the plane. Bar OA has mass $m = 10\ \text{(kg)}$ and length $L = 2\ \text{(m)}$. Block B has mass $m_B = 15\ \text{(kg)}$. The coefficient of kinetic friction between the block and the plane is $\mu_k = 0.2$. The drum has radius $r = 250\ \text{(mm)}$, and its mass and inertia are negligible.

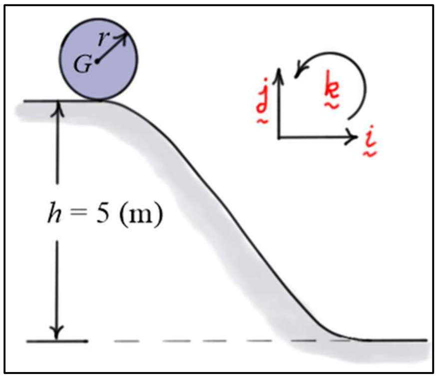


Given the system is released from rest at $\theta = 30\ \text{(deg)}$, find: a) $U_{1\to 2}$ the total work done on the system as it moves from $\theta = 30\ \text{(deg)}$ to $\theta = 0\ \text{(deg)}$, and b) $\underset{\sim}{\omega}_{OA}$ the angular velocity of OA when it reaches $\theta = 0$.

Answers:

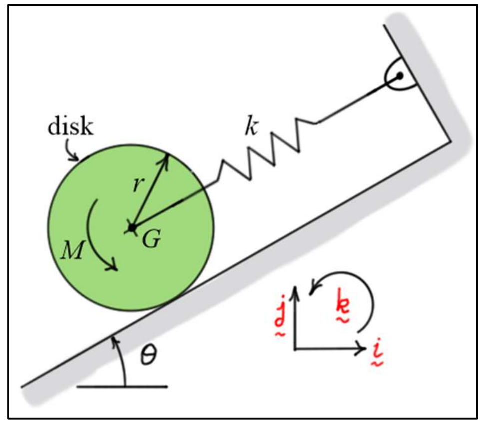
a) $U_{1\to 2} \approx 9.29\ \text{(N-m)}$; b) $\underset{\sim}{\omega}_{OA} \approx -1.14\,\underset{\sim}{k}\ \text{(rad/s)}$

3. The thin disk rolls without slipping down the $5\ \text{(m)}$ hill as shown. The disk has mass $m = 7\ \text{(kg)}$ and radius $r = 0.4\ \text{(m)}$. Given it is released from rest at the top of the hill, find $\underset{\sim}{\omega}_D$ the angular velocity of the disk when it reaches the bottom of the hill.



Answer:
$\underset{\sim}{\omega}_D \approx -20.2\,\underset{\sim}{k}\ \text{(rad/s)}$

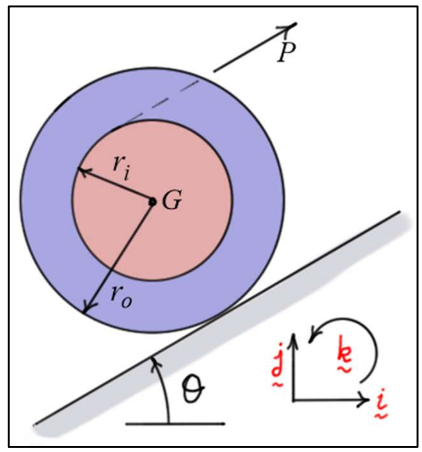
4. The 20-kg disk of radius $r = 0.2$ (m) is initially at rest and the spring is unstretched when the torque of $M = 30$ (N-m) is applied. The plane is inclined at an angle of $\theta = 30$ (deg), and the spring has stiffness $k = 150$ (N/m). Find: a) $U_{1\to2}$ the work done on the system as G moves down the plane 0.8 (m), and b) $\underset{\sim}{\omega}_D$ the angular velocity of the disk at the instant G arrives at that position. Assume that the disk rolls without slipping.



Answer:

a) $U_{1\to2} \approx 150$ (N-m); b) $\underset{\sim}{\omega}_D \approx 15.8\underset{\sim}{k}$ (rad/s)

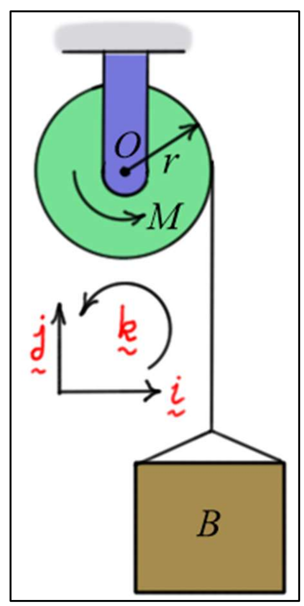
5. The 100 (kg) spool is pulled up the inclined plane ($\theta = 30$ (deg)) by a cable with tension force $P = 500$ (N). The spool starts from rest and rolls without slipping. The inner and outer radii of the spool are $r_i = 0.3$ (m) and $r_o = 0.4$ (m), and the radius of gyration of the spool about its mass center G is $k_G = 0.25$ (m). Find: a) $U_{1\to2}$ the work done on the spool as G moves a distance of 2 (m) up the inclined plane, and b) $\underset{\sim}{\omega}_S$ the angular velocity of the spool at the instant G arrives at the 2-meter position.



Answers:

a) $U_{1\to2} \approx 769$ (N-m); b) $\underset{\sim}{\omega}_S \approx -8.31\underset{\sim}{k}$ (rad/s)

6. A heavy bucket B weighing 1500 (lb) is lifted using a motor and pulley system as shown. The motor applies a constant torque of $M = 2000$ (ft-lb) to the pulley at O. The pulley has radius $r = 1.25$ (ft), radius of gyration about its center $k_O = 0.95$ (ft), and weight $W_p = 115$ (lb). The system starts from rest. Find: a) $U_{1\to2}$ the total work done on the system as the bucket is raised 10 (ft), and b) $\underset{\sim}{v}_B$ the velocity of the bucket when it reaches the 10-foot position.



Answers:

a) $U_{1\to2} \approx 1000$ (ft-lb); b) $\underset{\sim}{v}_B \approx 6.41\underset{\sim}{j}$ (ft/s)