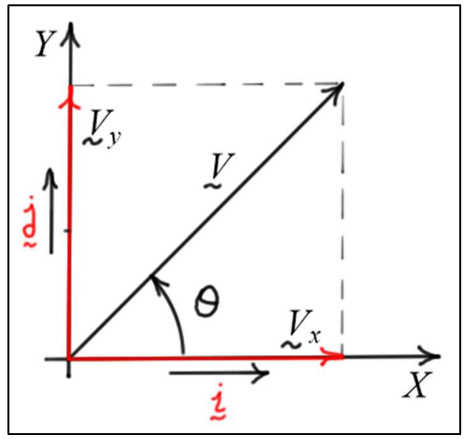
Elementary Statics
Vector Components and Vector Addition in Two Dimensions

Cartesian Components of Vectors in Two Dimensions

- Given the ***magnitude*** of a vector and the ***direction*** of the vector relative to a set of ***reference axes***, the vector can be expressed in terms of its ***components*** along those axes.



- For our convenience, it is usually beneficial to have the reference axes be ***mutually perpendicular***.
- In the diagram, $\underset{\sim}{V}_x$ and $\underset{\sim}{V}_y$ represent the components of the vector $\underset{\sim}{V}$ along the mutually perpendicular X and Y axes.
- The parallelogram formed by $\underset{\sim}{V}_x$ and $\underset{\sim}{V}_y$ is now a ***rectangle***, and the triangle formed by $\underset{\sim}{V}_x$ and $\underset{\sim}{V}_y$ is now a ***right triangle***.
- So, if the magnitude of the vector $\underset{\sim}{V}$ is $|\underset{\sim}{V}| = V$, we now have

$$\boxed{\underset{\sim}{V} = \underset{\sim}{V}_x + \underset{\sim}{V}_y = V\cos(\theta)\,\underset{\sim}{i} + V\sin(\theta)\,\underset{\sim}{j}}$$

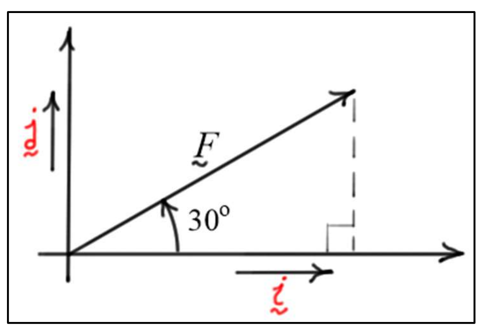
<u>Example #1</u>:

Given: A force $\underset{\sim}{F}$ has magnitude $\left|\underset{\sim}{F}\right| = F = 100 \text{ (lbs)}$ and angle $\theta = 30 \text{ (deg)}$.

Find: Express the force $\underset{\sim}{F}$ in terms of the unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$.

Solution:

$$\boxed{\underset{\sim}{F} = 100\cos(30)\,\underset{\sim}{i} + 100\sin(30)\,\underset{\sim}{j} \approx 86.6\,\underset{\sim}{i} + 50\,\underset{\sim}{j} \text{ (lb)}}$$

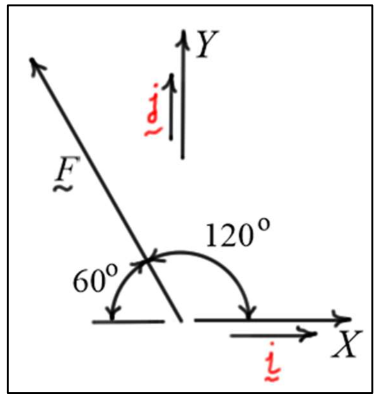


Example #2:

Given: A force $\underset{\sim}{F}$ has magnitude $|\underset{\sim}{F}| = 100$ (lbs) and angle $\theta = 120$ (deg).

Find: Express the force $\underset{\sim}{F}$ in terms of the unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$.

Solution:



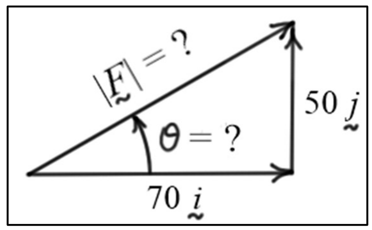
$$\begin{aligned} \underset{\sim}{F} &= 100\cos(120)\,\underset{\sim}{i} + 100\sin(120)\,\underset{\sim}{j} \\ &= -100\cos(60)\,\underset{\sim}{i} + 100\sin(60)\,\underset{\sim}{j} \\ &\approx -50\,\underset{\sim}{i} + 86.6\,\underset{\sim}{j} \text{ (lb)} \end{aligned}$$

Example #3:

Given: ***Force*** $\underset{\sim}{F} = 70\,\underset{\sim}{i} + 50\,\underset{\sim}{j}$ (lb).

Find: ***Magnitude*** and ***direction*** of $\underset{\sim}{F}$.

Solution:



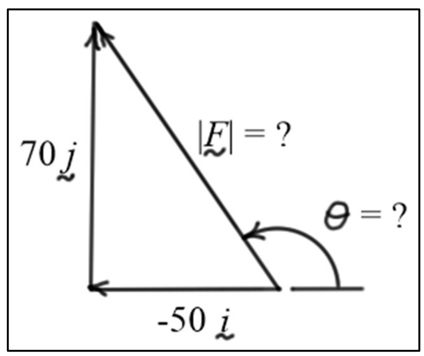
$$|\underset{\sim}{F}| = \sqrt{70^2 + 50^2} \approx 86.0 \text{ (lbs)}$$ and $$\theta = \tan^{-1}(50/70) \approx \begin{cases} 35.54 \text{ (deg)} \\ 0.6202 \text{ (rad)} \end{cases}$$

Example #4:

Given: ***Force*** $\underset{\sim}{F} = -50\,\underset{\sim}{i} + 70\,\underset{\sim}{j}$ (lb).

Find: ***Magnitude*** and ***direction*** of $\underset{\sim}{F}$.

Solution:



$$|\underset{\sim}{F}| = \sqrt{(-50)^2 + 70^2} = 86.0 \text{ (lb)}$$ $$\theta = \tan^{-1}(70/-50) = \begin{cases} -54.46 + 180 = 125.5 \text{ (deg)} \\ -0.9505 + \pi = 2.191 \text{ (rad)} \end{cases}$$

Notice that care must be taken to identify the correct quadrant when using the inverse tangent function. In this case, 180 degrees (or π radians) was added to the calculator result to find the correct result in the second quadrant.

Vector Addition using Cartesian Components in Two Dimensions

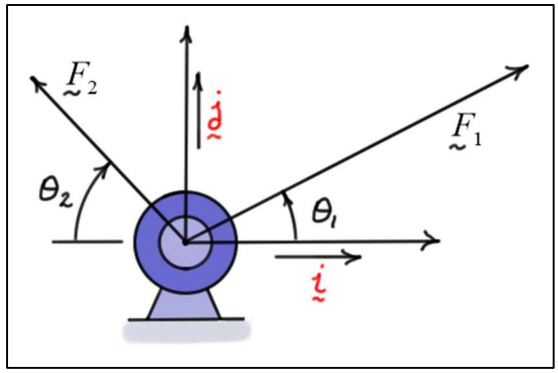
To ***add two*** or ***more vectors***, simply express them in terms of the same unit vectors, and then add ***like components***.

Example #5:

Given: ***Forces***

$$\left|\underset{\sim}{F}_1\right| = 150 \text{ (lb)},\ \theta_1 = 20 \text{ (deg)}$$

$$\left|\underset{\sim}{F}_2\right| = 100 \text{ (lb)},\ \theta_2 = 60 \text{ (deg)}$$



Find: a) ***Resultant force*** $\underset{\sim}{F}$ acting on the support in terms of the unit vectors shown.

b) ***Magnitude*** and ***direction*** of $\underset{\sim}{F}$.

Solution:

a) The total force is the vector sum of the two forces.

$$\underset{\sim}{F}_1 = 150\cos(20)\,\underset{\sim}{i} + 150\sin(20)\,\underset{\sim}{j} \approx 140.95\,\underset{\sim}{i} + 51.3\,\underset{\sim}{j} \text{ (lb)}$$

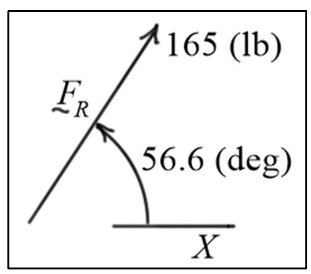
$$\underset{\sim}{F}_2 = -100\cos(60)\,\underset{\sim}{i} + 100\sin(60)\,\underset{\sim}{j} \approx -50\,\underset{\sim}{i} + 86.6\,\underset{\sim}{j} \text{ (lb)}$$

$$\boxed{\underset{\sim}{F} = \underset{\sim}{F}_1 + \underset{\sim}{F}_2 \approx (140.95 - 50)\,\underset{\sim}{i} + (51.3 + 86.6)\,\underset{\sim}{j} = 90.95\,\underset{\sim}{i} + 137.9\,\underset{\sim}{j} \text{ (lb)}}$$

b)

$$\boxed{\left|\underset{\sim}{F}\right| \approx \sqrt{90.95^2 + 137.9^2} \approx 165.2 \approx 165 \text{ (lb)}}$$

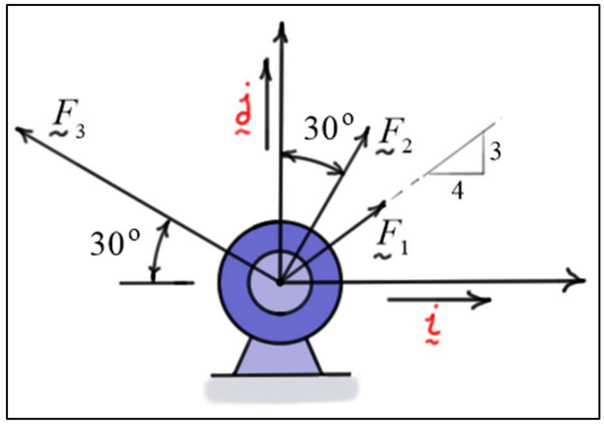
$$\boxed{\theta \approx \tan^{-1}\left(137.9/90.95\right) \approx \begin{cases} 56.6 \text{ (deg)} \\ 0.988 \text{ (rad)} \end{cases}}$$



Example #6: (3 forces)

Given: $|\underset{\sim}{F}_1| = 50$ (lb); $|\underset{\sim}{F}_2| = 75$ (lb), $|\underset{\sim}{F}_3| = 150$ (lb)

- all directions are as shown in the diagram



Find: a) ***Resultant force*** $\underset{\sim}{F}_R$ acting on the support in terms of the unit vectors shown.

b) ***Magnitude*** and ***direction*** of $\underset{\sim}{F}_R$.

Solution:

a) $\underset{\sim}{F}_1 = 50\left(\frac{4}{5}\underset{\sim}{i} + \frac{3}{5}\underset{\sim}{j}\right) = 40\,\underset{\sim}{i} + 30\,\underset{\sim}{j}$ (lb)

$\underset{\sim}{F}_2 = 75\left(\sin(30)\,\underset{\sim}{i} + \cos(30)\,\underset{\sim}{j}\right) \approx 37.5\,\underset{\sim}{i} + 64.9519\,\underset{\sim}{j}$ (lb)

$\underset{\sim}{F}_3 = 150\left(-\cos(30)\,\underset{\sim}{i} + \sin(30)\,\underset{\sim}{j}\right) \approx -129.9\,\underset{\sim}{i} + 75\,\underset{\sim}{j}$ (lb)

$$\underset{\sim}{F}_R \approx (40 + 37.5 - 129.9)\,\underset{\sim}{i} + (30 + 64.95 + 75)\,\underset{\sim}{j}$$

$$\approx -52.4038\,\underset{\sim}{i} + 169.952\,\underset{\sim}{j}$$

$$\Rightarrow \boxed{\underset{\sim}{F}_R \approx -52.4\,\underset{\sim}{i} + 170\,\underset{\sim}{j}}$$

b) $$\boxed{|\underset{\sim}{F}_R| \approx \sqrt{(-52.4038)^2 + (169.952)^2} \approx 177.848 \approx 178 \text{ (lb)}}$$

$$\boxed{\theta \approx \tan^{-1}\left(\frac{169.952}{-52.4038}\right) \approx -72.86 + 180 \approx 107 \text{ (deg)}}$$