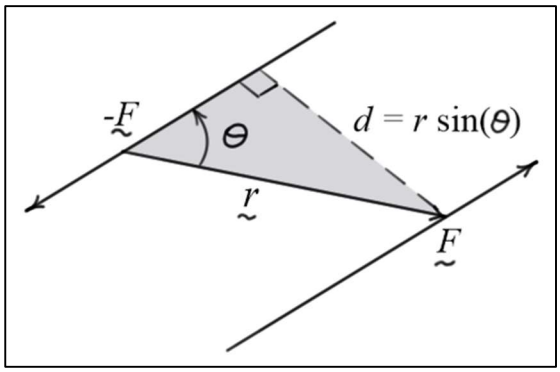
Elementary Statics
Force Couples



- ***Definition***: A ***force couple*** is a pair of ***equal and opposite forces*** whose lines of action are separated by some distance, *d*.
- A force couple is simply referred to as a ***couple***.
- As the forces are ***equal and opposite***, the ***resultant*** of the two forces is ***zero***.
- It can be shown that the ***sum*** of the ***moments*** of the two forces about ***any point*** is

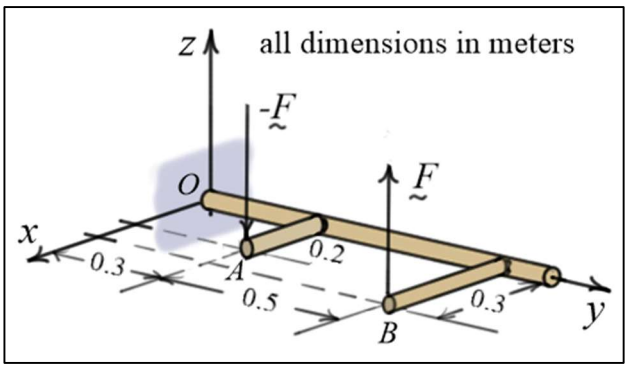
  $$\boxed{\underset{\sim}{M} = \underset{\sim}{r} \times \underset{\sim}{F}}$$

  Here, $\underset{\sim}{r}$ is the ***position vector*** from ***any point*** on the ***line of action*** of one of the forces to ***any point*** on the ***line of action*** of the other force.
- The ***magnitude*** of the moment (or torque) is $\boxed{M = |\underset{\sim}{M}| = F\,r\,\sin(\theta) = F\,d}$.
- As before, the ***direction*** of the moment is given by the ***right-hand rule*** and is ***perpendicular*** to the ***plane*** formed by the two forces.
- The ***moment*** of the ***couple*** is often referred to as a ***couple moment***.
- Because the ***moment*** of a couple is the ***same*** about all points, it is a ***free vector***.

Resultant of a Set of Couples

- The ***resultant force*** of a set of couples is ***zero***.
- The ***resultant moment*** of a set of couples is the ***vector sum*** of the ***couple moments***.

Example #1:



Given: $\left|\underset{\sim}{F}\right| = F = 80$ (N)

Find: $\underset{\sim}{M}_C$ the moment (or torque) of the couple

Solution:

$$\underset{\sim}{M}_C = \underset{\sim}{r}_{B/A} \times \underset{\sim}{F} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 80 \end{vmatrix} = \left(0.5(80)\right)\underset{\sim}{i} - \left(0.1(80)\right)\underset{\sim}{j} \quad \Rightarrow \boxed{\underset{\sim}{M}_C = 40\,\underset{\sim}{i} - 8\,\underset{\sim}{j}\ \text{(N-m)}}$$

$$\boxed{\left|\underset{\sim}{M}_C\right| = \sqrt{40^2 + 8^2} \approx 40.8\ \text{(N-m)}}$$

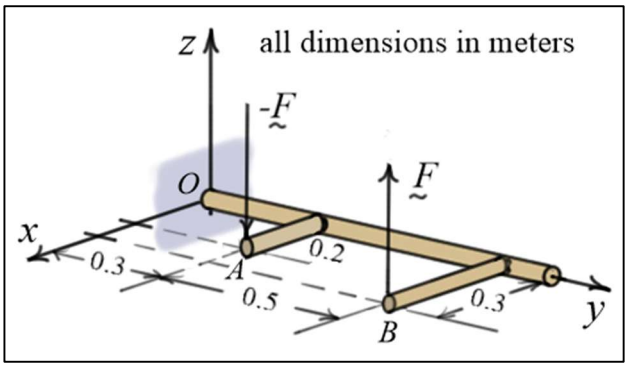
Check: Recall the moment of a couple is the same about all points. So, we can also write

$$\underset{\sim}{M}_C = \left(\underset{\sim}{r}_{B/O} \times \underset{\sim}{F}\right) + \left(\underset{\sim}{r}_{A/O} \times -\underset{\sim}{F}\right) = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 0.3 & 0.8 & 0 \\ 0 & 0 & 80 \end{vmatrix} + \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 0.2 & 0.3 & 0 \\ 0 & 0 & -80 \end{vmatrix}$$

$$= \left(0.8(80) - 0.3(80)\right)\underset{\sim}{i} - \left(0.3(80) - 0.2(80)\right)\underset{\sim}{j}$$

$$\Rightarrow \boxed{\underset{\sim}{M}_C = 40\,\underset{\sim}{i} - 8\,\underset{\sim}{j}\ \text{(N-m)}}\ \text{(same result)}$$

Example #2:



Given: $\left|\underset{\sim}{M}_C\right| = 50$ (N-m)

Find: $\underset{\sim}{F}$

Solution:

$$\underset{\sim}{M}_C = \underset{\sim}{r}_{B/A} \times \underset{\sim}{F} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & F \end{vmatrix} = \left(0.5F\right)\underset{\sim}{i} - \left(0.1F\right)\underset{\sim}{j}$$

$$\left|\underset{\sim}{M}_C\right| = \sqrt{0.5^2 F^2 + 0.1^2 F^2} = F\sqrt{0.5^2 + 0.1^2} \quad \Rightarrow \boxed{F = \frac{50}{\sqrt{0.5^2 + 0.1^2}} \approx 98.1\ \text{(N)}}$$