ME 2560 Statics
Truss Analysis – Two Dimensional

Definition and Observations:

- A ***simple truss*** is a ***load bearing structure*** which consists of a collection of ***two-force members*** connected by ***smooth*** (frictionless) ***pins***. These pin connections are called ***joints***.
- For the members of the truss to be ***two-force members***, all ***external loads*** must be located at the ***joints***. As the weights of each of the members act at their individual centers of gravity, we will ***neglect their weights*** in the analysis. As a first approximation, weights can be included by applying ***half*** the ***weight*** of each member to the joints that member connects.
- Each member of the truss is either in a state of ***tension*** or ***compression***, or it carries ***no load***. Members that carry ***no load*** are referred to as ***zero-force members***.

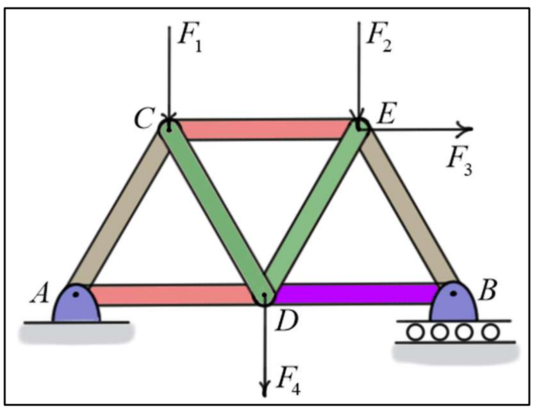


Fig. 1 Seven Member Truss

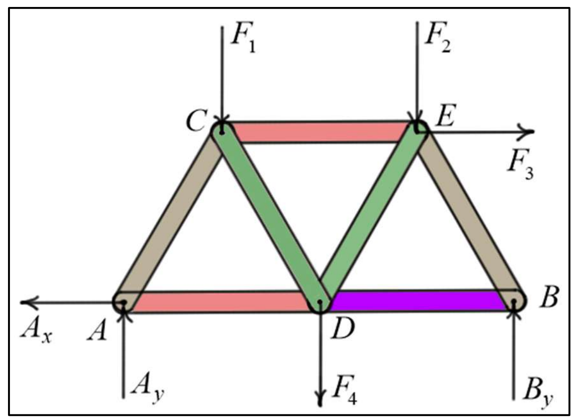


Fig. 2 Free Body Diagram of Truss

External Support Forces

- The ***external support reaction forces*** holding the truss in equilibrium are found by treating the entire truss as a rigid body. First, ***draw*** a ***free body diagram*** of the truss (Fig. 2), then ***write*** and ***solve*** the equations of equilibrium.

Internal Member Forces

- The ***forces*** in each of the ***members*** of the truss are found by applying the ***method*** of ***joints*** (or ***pins***), the ***method*** of ***sections***, or a ***combination*** of ***both***.

Internal Member Forces using the Method of Joints

- ***Draw free body diagrams*** of the ***pins***, then ***write*** and ***solve*** the equilibrium equations. As all the ***forces*** on a pin ***radiate*** from that point, ***only force equations*** are required.
- ***Tensile*** forces ***act away*** from the pins. ***Compressive*** forces ***act towards*** the pins. All member forces in Fig. 3 are drawn as tensile forces.

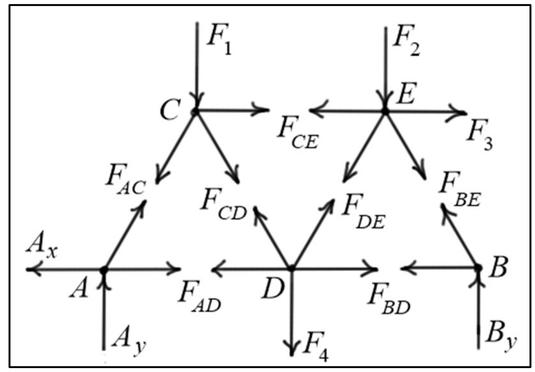


Fig. 3 Free Body Diagrams of all Pins

- If this convention is followed when solving for the unknown member forces, all ***positive results*** indicate ***tensile forces***, and all ***negative results*** indicate ***compressive forces***.
- ***Before*** doing this analysis, it may be ***helpful*** to first find the external support forces.

Internal Member Forces using the Method of Sections

- ***Isolate*** (cut-away) any ***section*** of the truss and treat it as a rigid body. ***Draw*** a ***free body diagram*** of that section, then ***write*** and ***solve*** the equations of equilibrium.
- An important part of this strategy is to ***identify sections*** of the truss that have only ***three unknown forces*** acting on them. To find such a section it may be necessary to first find the external support forces. In the section shown in Fig. 5, if the external support forces A_x and A_y are known, only three unknown member forces remain on that section.
- As before, ***tensile*** member forces ***act away*** from the pins, and ***compressive*** member forces ***act towards*** the pins.

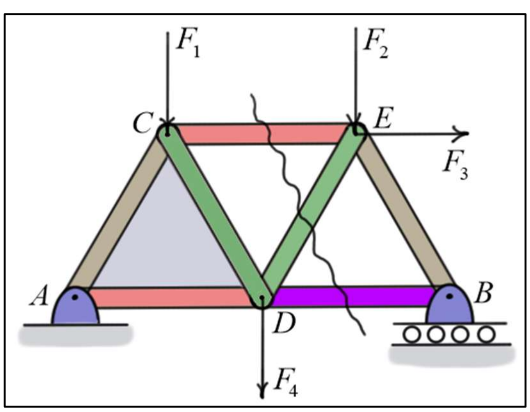


Fig. 4 Cut to Isolate Section ACD

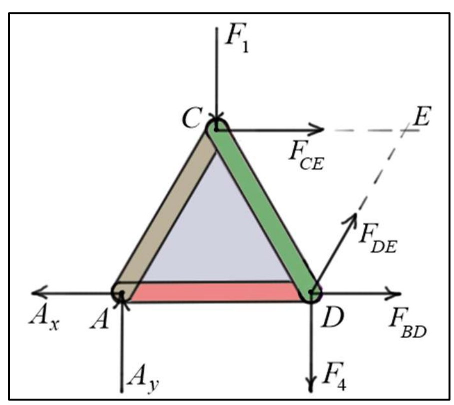
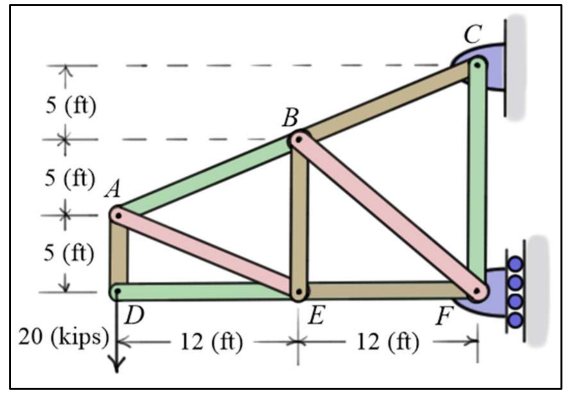


Fig. 5 Free Body Diagram of Section ACD

<u>Example #1</u>:

Given: Simple truss loaded as shown

Find: All member and support reaction forces.



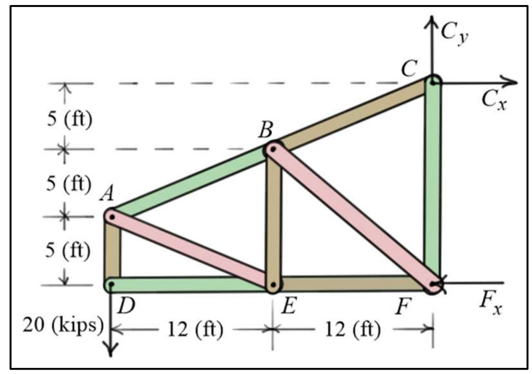
Solution:

External Support Forces:

$$\circlearrowleft\!+\ \sum M_C = 24(20) - 15F_x = 0 \quad \Rightarrow \boxed{F_x = 32 \text{ (kips)}}$$

$$\circlearrowleft\!+\ \sum M_F = 24(20) - 15C_x = 0 \quad \Rightarrow \boxed{C_x = 32 \text{ (kips)}}$$

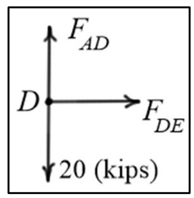
$$\sum F_y = C_y - 20 = 0 \quad \Rightarrow \boxed{C_y = 20 \text{ (kips)}}$$



Free Body Diagram of Entire Truss

Member Forces: T(tension), C(compression)

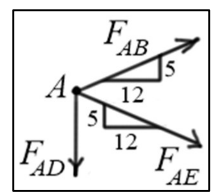
Pin D:



$$\boxed{\sum F_x = F_{DE} = 0}$$

$$\sum F_y = F_{AD} - 20 = 0 \quad \Rightarrow \boxed{F_{AD} = 20 \text{ (kips) T}}$$

Pin A:



$$\sum F_y = \tfrac{5}{13}F_{AB} - \tfrac{5}{13}F_{AE} - F_{AD} = 0 \quad \Rightarrow \boxed{F_{AB} - F_{AE} = \tfrac{13}{5}F_{AD} = 52}$$

$$\sum F_x = \tfrac{12}{13}F_{AB} + \tfrac{12}{13}F_{AE} = 0 \quad \Rightarrow \boxed{F_{AE} = -F_{AB}}$$

Solving gives: $\boxed{F_{AB} = 52/2 = 26 \text{ (kips) T}}$, $\boxed{F_{AE} = -26 = 26 \text{ (kips) C}}$

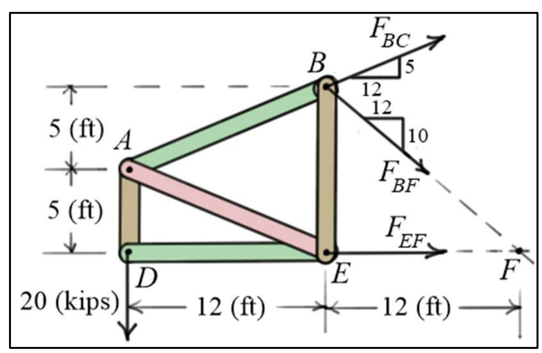
Section $ABED$:

$$\circlearrowleft\!+\ \sum M_B = 10F_{EF} + 12(20) = 0 \quad \Rightarrow \boxed{F_{EF} = 24 \text{ (kips) C}}$$

$$\circlearrowleft\!+\ \sum M_F = 24(20) - 10\left(\tfrac{12}{13}F_{BC}\right) - 12\left(\tfrac{5}{13}F_{BC}\right) = 0$$

$$\Rightarrow \tfrac{180}{13}F_{BC} = 480$$

$$\Rightarrow \boxed{F_{BC} \approx 34.666 \approx 34.7 \text{ (kips) T}}$$

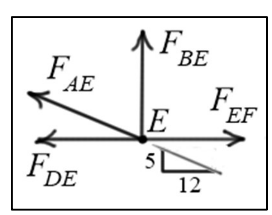


$$\sum F_y = \tfrac{5}{13}F_{BC} - \tfrac{10}{\sqrt{244}}F_{BF} - 20 = 0$$

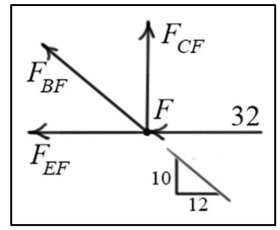
$$\Rightarrow F_{BF} = \tfrac{\sqrt{244}}{10}\left(\tfrac{5}{13}F_{BC} - 20\right) \approx -10.414 \quad \Rightarrow \boxed{F_{BF} \approx 10.4 \text{ (kips) C}}$$

Pin E:

$$\sum F_y = F_{BE} + \tfrac{5}{13}F_{AE} = 0 \quad \Rightarrow \boxed{F_{BE} = -\tfrac{5}{13}(-26) = 10 = 10 \text{ (kips) T}}$$



Pin F:



$$\sum F_y = F_{CF} + \tfrac{10}{\sqrt{244}} F_{BF} = 0 \quad \Rightarrow F_{CF} = -\tfrac{10}{\sqrt{244}}\left(\tfrac{\sqrt{244}}{10}\left(\tfrac{5}{13}F_{BC} - 20\right)\right)$$

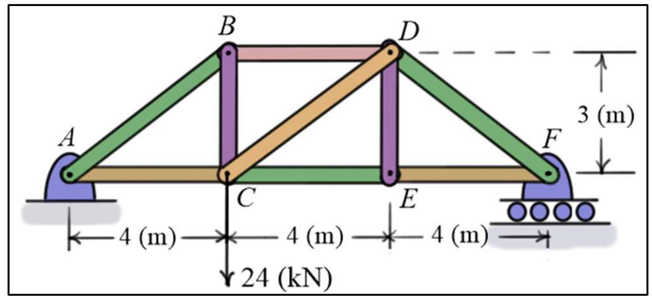
$$= 20 - \tfrac{5}{13}\left(\tfrac{13}{180}\right)480 \quad \Rightarrow \boxed{F_{CF} \approx 6.67 \text{ (kips) T}}$$

Note: Although the external support forces acting on this truss were found prior to the member forces, the values of those forces were not needed for calculation of most of the member forces.

<u>Example #2</u>:

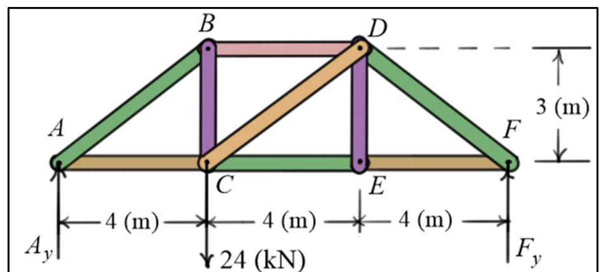
Given: Simple truss loaded as shown.

Find: All external support and member forces.



Solution:

External Support Forces: (entire truss)



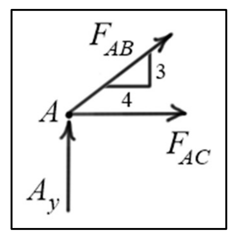
Free Body Diagram of Entire Truss

$\boxed{\sum F_x = 0}$ … no forces in the x direction

$$\circlearrowleft\!\!+ \sum M_A = 12F_y - 4(24) = 0 \quad \Rightarrow \boxed{F_y = 8 \text{ (kN)}}$$

$$\sum F_y = A_y + F_y - 24 = 0 \quad \Rightarrow \boxed{A_y = 16 \text{ (kN)}}$$

Pin A:

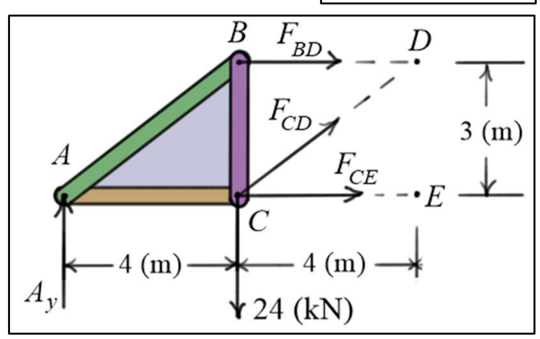


$$\sum F_y = A_y + \tfrac{3}{5}F_{AB} = 0$$

$$\Rightarrow F_{AB} = -\tfrac{5}{3}A_y = -\tfrac{5}{3}(16) \approx -26.6667 \quad \Rightarrow \boxed{F_{AB} \approx 26.7 \text{ (kN) C}}$$

$$\sum F_x = F_{AC} + \tfrac{4}{5}F_{AB} = 0 \quad \Rightarrow F_{AC} = -\tfrac{4}{5}F_{AB} \approx 21.333 \quad \Rightarrow \boxed{F_{AC} \approx 21.3 \text{ (kN) T}}$$

Section ABC:



$$\sum F_y = A_y - 24 + \tfrac{3}{5}F_{CD} = 0$$

$$\Rightarrow F_{CD} = \tfrac{5}{3}\left(24 - A_y\right) = \tfrac{5}{3}(8)$$

$$\Rightarrow \boxed{F_{CD} \approx 13.3333 \approx 13.3 \text{ (kN) T}}$$

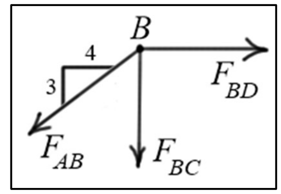
$$\circlearrowleft\!\!+ \sum M_D = 3F_{CE} + 4(24) - 8A_y = 0$$

$$\Rightarrow F_{CE} = \tfrac{1}{3}\left(8A_y - 4(24)\right) \approx 10.6667 \quad \Rightarrow \boxed{F_{CE} \approx 10.7 \text{ (kN) T}}$$

$$\sum M_C = -4A_y - 3F_{BD} = 0 \quad \Rightarrow F_{BD} = -\tfrac{4}{3}A_y \approx -21.3333$$

$$\Rightarrow \boxed{F_{BD} \approx 21.3 \text{ (kN) C}}$$

Pin *B*:

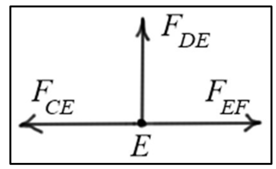


$$\sum F_y = -F_{BC} - \tfrac{3}{5}F_{AB} = 0 \quad \Rightarrow F_{BC} = -\tfrac{3}{5}F_{AB} \approx -\tfrac{3}{5}(-26.6666)$$

$$\Rightarrow \boxed{F_{BC} = 16 \text{ (kN) T}}$$

Check: $\boxed{\sum F_x = F_{BD} - \tfrac{4}{5}F_{AB} = -21.3333 - \tfrac{4}{5}(-26.6666) \approx 0}$

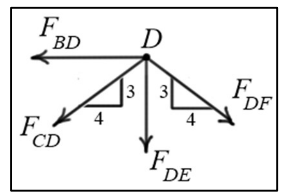
Pin *E*:



$$\sum F_x = F_{EF} - F_{CE} = 0 \quad \Rightarrow \boxed{F_{EF} = F_{CE} \approx 10.6666 \approx 10.7 \text{ (kN) T}}$$

$\boxed{\sum F_y = F_{DE} = 0}$ (*DE* is a zero-force member)

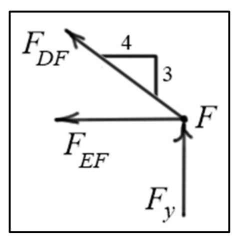
Pin *D*:



$$\sum F_y = -\tfrac{3}{5}F_{CD} - \underbrace{F_{DE}}_{\text{zero}} - \tfrac{3}{5}F_{DF} = 0 \quad \Rightarrow \boxed{F_{DF} = -F_{CD} \approx 13.3 \text{ (kN) C}}$$

Check: $\begin{cases} \sum F_x = \tfrac{4}{5}F_{DF} - \tfrac{4}{5}F_{CD} - F_{BD} \\ \qquad \approx \tfrac{4}{5}(-13.3333) - \tfrac{4}{5}(13.3333) - (-21.3333) \approx -21.3333 + 21.3333 \\ \qquad \Rightarrow \boxed{\sum F_x \approx 0} \end{cases}$

Pin *F*: (check)



$$\boxed{\sum F_x = -F_{EF} - \tfrac{4}{5}F_{DF} \approx -10.6666 - \tfrac{4}{5}(-13.3333) \approx 0}$$

$$\boxed{\sum F_y = F_y + \tfrac{3}{5}F_{DF} \approx 8 + \tfrac{3}{5}(-13.3333) \approx 8 - 7.99999 \approx 0}$$

Note: In this second example, it was necessary to find the external support forces before proceeding with the calculation of the member forces.