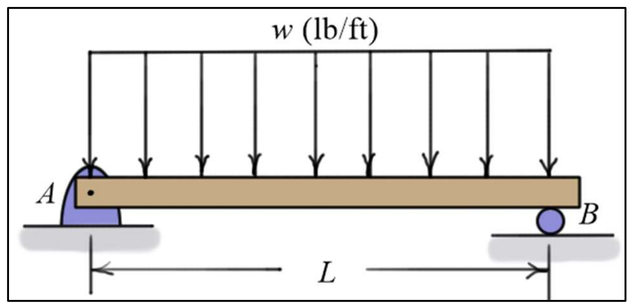
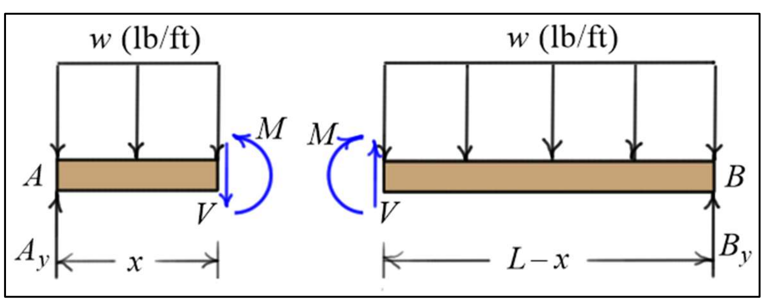
Elementary Statics
Internal Forces in Structural Members

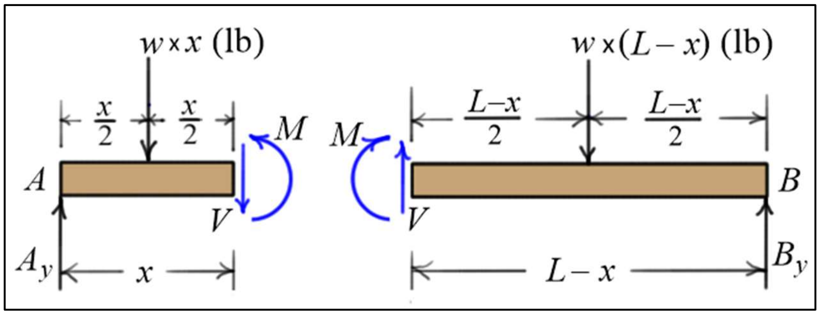
- To date, we have used the ***equations of equilibrium*** to find the ***external forces*** acting on the ***members*** of simple structures.
- The next step in the ***design*** of those structures is to ***size*** the ***members*** so they will ***survive*** under the action of these forces.
- To do this, we must be able to ***calculate*** the ***internal forces*** and ***torques*** and the corresponding ***stresses*** and ***strains*** in each of the members.
- An essential part of this design process includes choosing the ***materials*** used for the construction (steel, concrete, aluminum, graphite, ceramic, etc.).
- Consider a ***simply supported beam*** with a ***constant distributed load*** as shown. By replacing the distributed load by a ***single concentrated load***, the ***support forces*** at ends A and B can be found.



- To find the ***internal forces*** and ***moments*** acting on any ***cross section*** of the beam, it is ***cut*** at that point to ***expose*** them. The diagrams below show the beam cut at a distance x from the left end. As the diagram indicates, the beam experiences a ***shear force*** V and a ***bending moment*** M at that point. Note that they are shown equal and opposite on the two sections in compliance with Newton's third law.

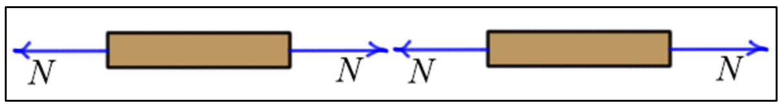


- The ***shear force*** V and ***bending moment*** M can be found by writing the ***equilibrium equations*** for either of the two free body diagrams. The ***distributed loads*** over each section can be replaced by single concentrated load on that section. See the diagram below.

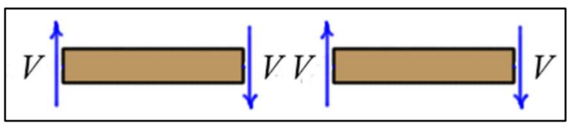


- Under ***general two-dimensional loading***, structural members will experience ***shear forces*** and ***bending moments*** as mentioned above as well as ***axial forces*** (tension or compression).
- By convention, the following diagrams illustrate ***positive axial forces***, ***positive shear forces***, and ***positive bending moments***.

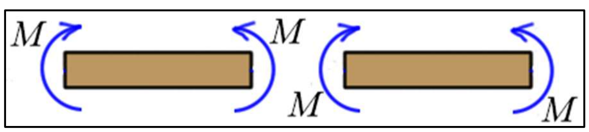
Positive axial force:
(tension)



Positive shear force:



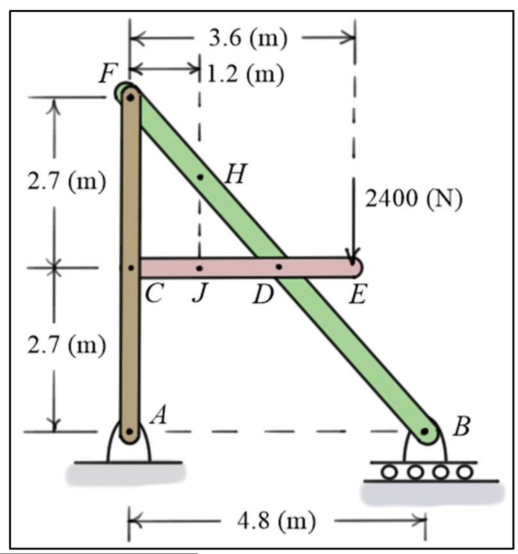
Positive bending moment:



Example:

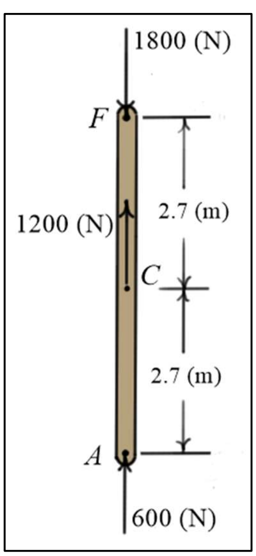
Given: Frame loaded as shown. Neglect member weights.

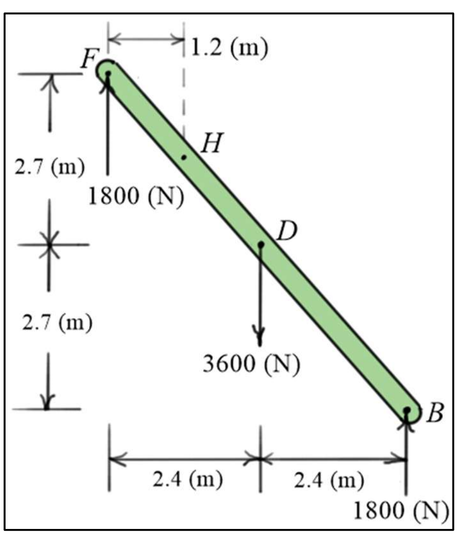
Find: Internal forces at H and J

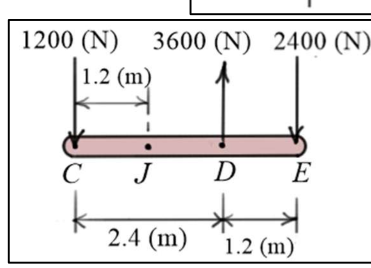


Solution:

The following results were found in previous notes.

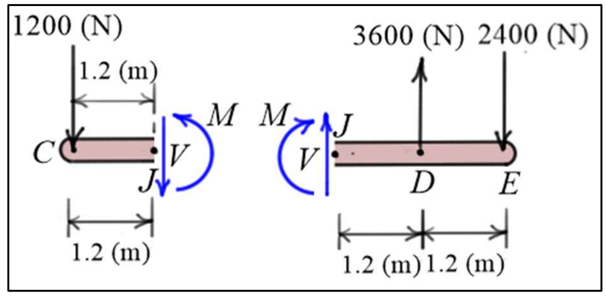






Member *CDE*:

The free body diagrams of the left and right ends of member *CDE* are shown in the diagram. The member has been cut at *J* exposing the internal forces and moments at that point. Note there are no horizontal forces on the member, so the axial force at the cut has been excluded.



Left End:

$$\circlearrowleft\!+\ \sum M_J = M + 1.2(1200) = 0 \quad \Rightarrow \boxed{M = -1440 \text{ (N-m)}} \quad \Rightarrow \boxed{M = 1440 \text{ (N-m)}} \circlearrowright$$

$$\sum F_y = -V - 1200 = 0 \quad \Rightarrow \boxed{V = -1200 \text{(N)}} \quad \Rightarrow \boxed{V = 1200 \text{(N)}} \uparrow$$

Right End: (check)

$$\circlearrowleft\!+\ \sum M_J = -M + 1.2(3600) - 2.4(2400) = 0 \quad \Rightarrow \boxed{M = -1440 \text{ (N-m)}}$$

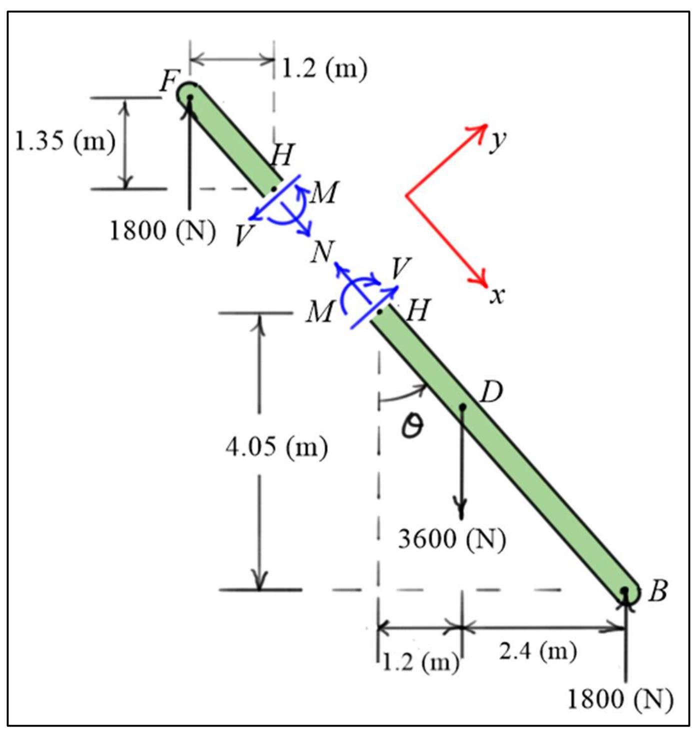
$$\Rightarrow \boxed{M = 1440 \text{ (N-m)}} \circlearrowleft \quad \checkmark$$

$$\sum F_y = V + 3600 - 2400 = 0 \quad \Rightarrow \boxed{V = -1200 \text{ (N)}} \quad \Rightarrow \boxed{V = 1200 \text{ (N)}} \downarrow \checkmark$$

Note the shear force *V* and bending moment *M* are equal and opposite on the cut faces.

Member *BDF*:

The free body diagrams of the upper and lower portions of member *BDF* are shown in the diagram. Note the internal forces and moments are drawn along and perpendicular to the member. In this case the member experiences a normal force *N*, a shear force *V*, and a bending moment *M* on the cut section.



Geometry:

$$\boxed{\theta = \tan^{-1}\left(\tfrac{3.6}{4.05}\right) \approx 41.6335 \approx 41.6 \text{ (deg)}}$$

Upper End:

$$\sum F_x = N - 1800\cos(\theta) = 0 \quad \Rightarrow \boxed{N = 1800\cos(\theta) \approx 1345.34 \approx 1345 \text{ (N)}}$$

$$\sum F_y = -V + 1800\sin(\theta) = 0 \quad \Rightarrow \boxed{V = 1800\sin(\theta) \approx 1195.85 \approx 1196 \text{ (N)}}$$

$$\circlearrowleft\!\!+ \sum M_H = M - 1.2(1800) = 0 \quad \Rightarrow \boxed{M \approx 2160 \text{ (N)}} \quad \Rightarrow \boxed{M \approx 2160 \text{ (N)}}$$

Lower End: (check)

$$\sum F_x = -N + 3600\cos(\theta) - 1800\cos(\theta) \quad \Rightarrow \boxed{N = 1800\cos(\theta) \approx 1345 \text{ (N)}} \checkmark$$

$$\sum F_y = V - 3600\sin(\theta) + 1800\sin(\theta) = 0 \quad \Rightarrow \boxed{V = 1800\sin(\theta) \approx 1196\text{(N)}} \checkmark$$

$$\circlearrowleft\!\!+ \sum M_H = -M - 1.2(3600) + 3.6(1800) = 0 \quad \boxed{M = 1.2(1800) = 2160 \text{ (N)}} \checkmark$$

As expected, the normal force N, shear force V, and bending moment M are all equal and opposite on the cut faces.