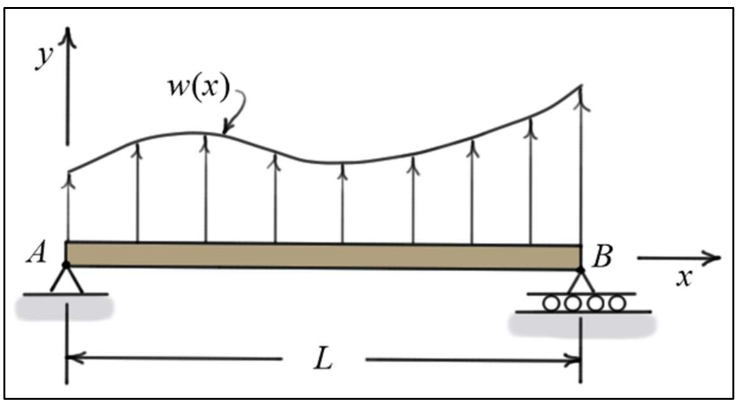
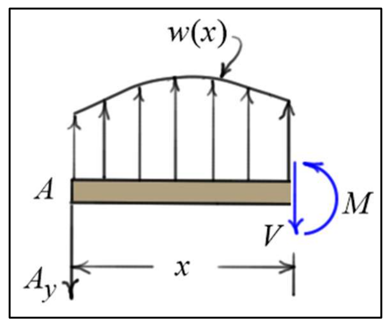
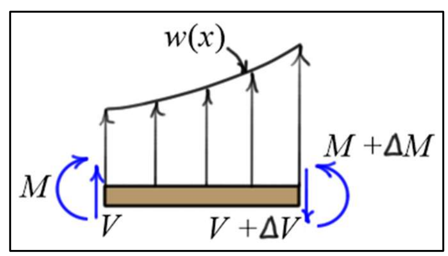
Elementary Statics
Shear Force and Bending Moment Diagrams

- Previously, we found the ***shear force*** and ***bending moment*** at any location along a structural member. The next step in the ***design*** of the member is to determine the ***locations*** of ***maximum shear force*** and ***maximum bending moment***.
- The ***largest stresses*** within the structural member are at ***these locations***.
- To this end, we ***plot*** the shear force and bending moment ***over the entire length*** of the structural member. These plots are called ***shear force*** and ***bending moment diagrams***.
- Consider a ***simply supported beam*** with a ***distributed load*** $w(x)$ (positive upward). As before, we find the internal shear force and bending moment by cutting the beam at some point. But now, we find the shear force and bending moment as a ***function*** of the cutting distance x.





- By analyzing a ***differential segment*** of the beam, it can be shown that the ***load intensity***, ***shear force***, and ***bending moment*** along the beam are related as follows.

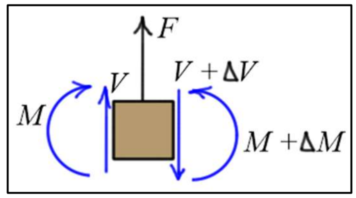


$$\frac{dV}{dx} = w(x) \qquad \Delta V = \int w(x)\,dx \quad \begin{cases} \text{Change in shear force =} \\ \text{Area under the load diagram} \end{cases}$$

$$\frac{dM}{dx} = V(x) \qquad \Delta M = \int V(x)\,dx \quad \begin{cases} \text{Change in bending moment =} \\ \text{Area under the shear diagram} \end{cases}$$

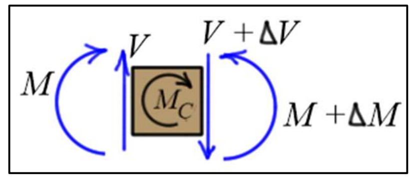
- The above equations are ***valid*** over segments of the beam where there are ***no concentrated forces*** or ***concentrated moments*** (couples). If there are concentrated forces or moments consider the following.

- When crossing the location of a ***concentrated upward load***, the shear force ***increases*** by that amount. When crossing the location of a ***concentrated downward load***, the shear force ***decreases*** by that amount.



$$+\downarrow \sum F = V + \Delta V - V - F = 0 \quad \Rightarrow \boxed{\Delta V = F}$$

- When crossing the location of a ***concentrated clockwise moment***, the bending moment ***increases*** by that amount. When crossing the location of a ***concentrated counterclockwise moment***, the bending moment ***decreases*** by that amount.

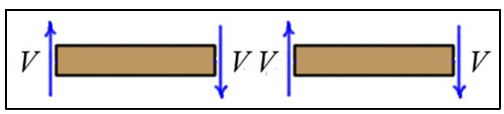


$$\circlearrowright\!+ \sum M = M + \Delta M - M - M_C = 0 \quad \Rightarrow \boxed{\Delta M = M_C}$$

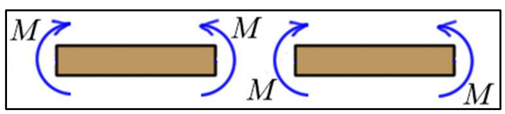
<u>Sign Conventions</u>

- These notes assume the ***load intensity*** $w(x)$ is ***positive upward***, and the internal ***shear force*** and ***bending moment*** are positive as shown below.

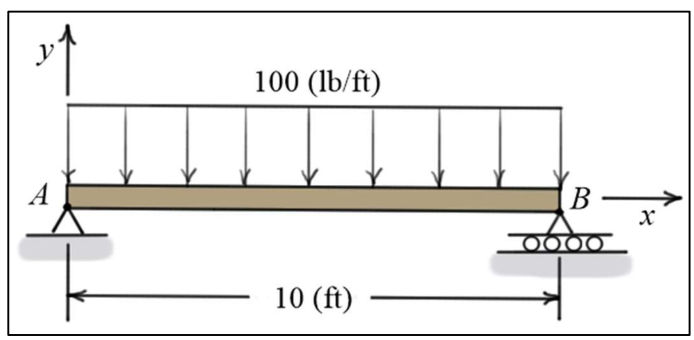
Positive shear force:



Positive bending moment:



<u>Example #1</u>:



Given: Beam loaded as shown

<u>Find</u>: (a) shear force diagram

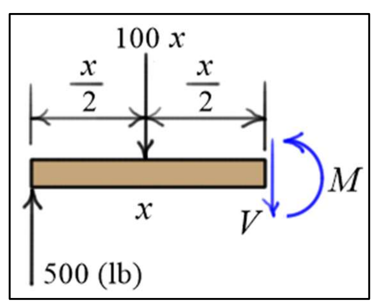
(b) bending moment diagram

(c) maximum bending moment

<u>Solution</u>:

By symmetry, the vertical loads at both ends A and B are

$$\boxed{A_y = B_y = \tfrac{1}{2}(10)(100) = 500 \text{ (lb)}}$$



a) Using a ***free body diagram*** of the left end of the beam:

$$+\downarrow \sum F = V + 100x - 500 = 0 \quad \Rightarrow \boxed{V(x) = 500 - 100x \;\; \text{(lb)}}$$

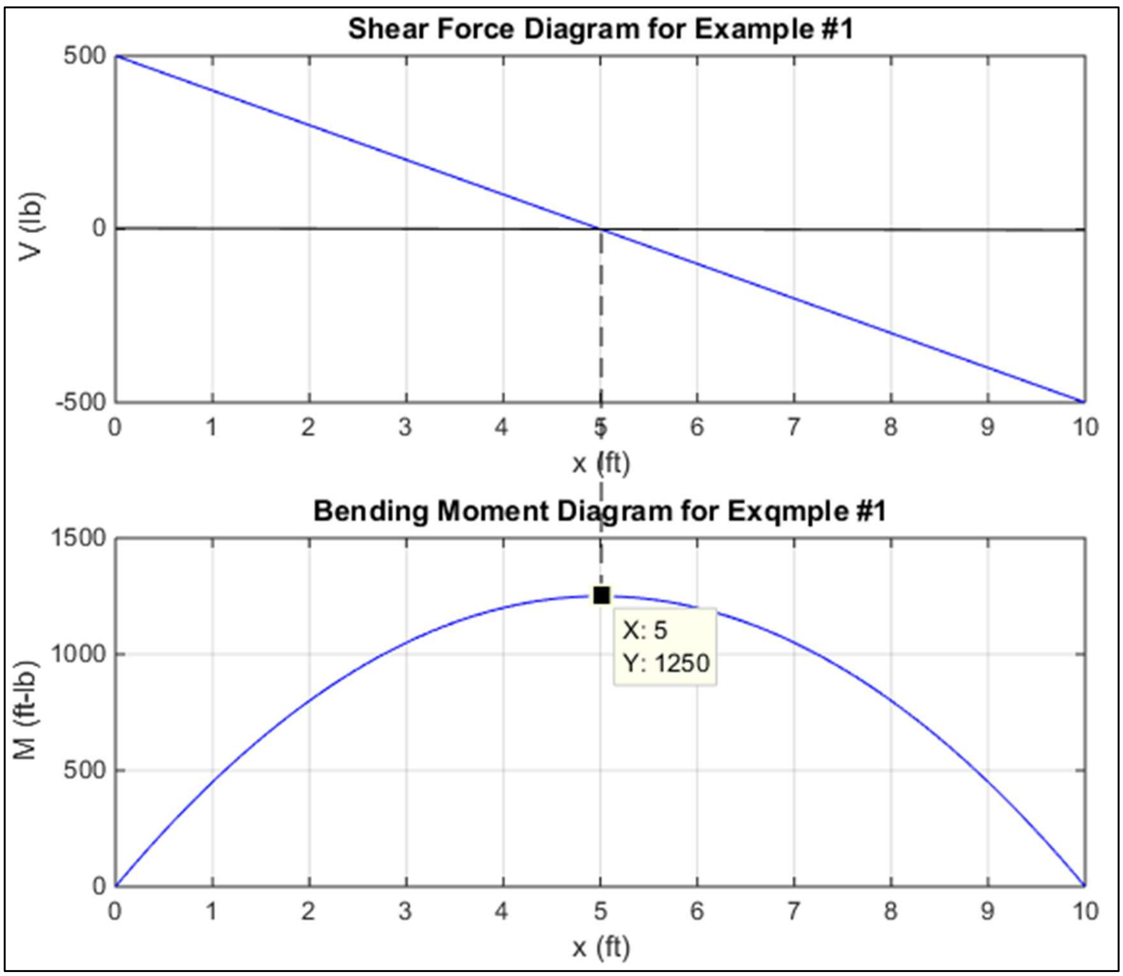
b) Again, using the ***free body diagram***,

$$\circlearrowright\!+ \sum M_{\text{cut}} = M + (100x)\cdot\left(\tfrac{x}{2}\right) - 500x = 0 \quad \Rightarrow \boxed{M(x) = 500x - 50x^2 \;\; \text{(ft-lb)}}$$

c) Because, $dM/dx = V(x)$, a ***local maximum bending moment*** will occur where $V = 0$. In this case, this occurs at $x = 5$ (ft) where the bending moment is $\boxed{M(5) = 1250 \text{ (ft-lb)}}$. In this case, this is also the ***global maximum moment***, that is the maximum moment along the beam.

Notes:

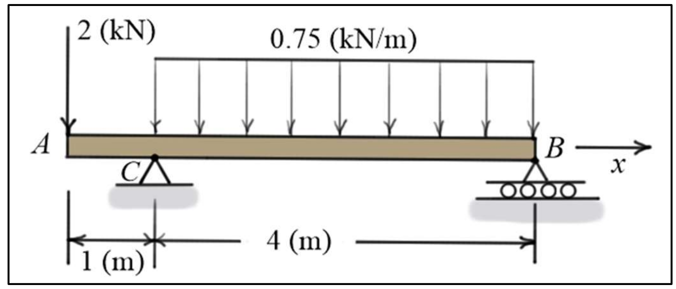
1. The ***shear diagram varies linearly*** in the range $(0 \le x \le 10)$. The ***slope*** is -100 (lb/ft) as indicated by the ***loading function***.
2. The ***moment diagram varies quadratically*** in the range $(0 \le x \le 10)$ starting with a ***positive slope*** and ***transitioning*** to a ***negative slope*** as indicated by the ***shear diagram***.
3. The ***change*** in the ***shear force*** over the range $(0 \le x \le 10)$ is equal to the ***area under the load diagram*** over this range. $\boxed{\Delta V = (-100)(10) = -1000 \text{ (lb)}}$
4. The ***change*** in ***bending moment*** over the range $(0 \le x \le 5)$ is the ***area under the shear diagram*** over this range. $\boxed{M_{\text{max}} = \tfrac{1}{2}(5)(500) = 1250 \text{ (ft-lb)}}$



Example #2:

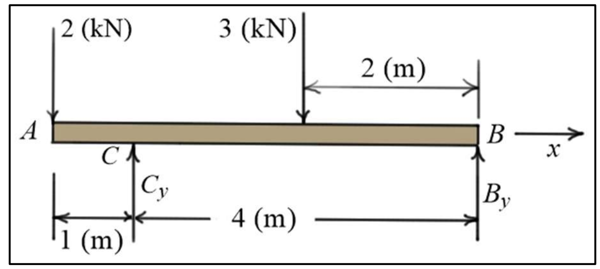
Given: Beam loaded as shown

Find: Shear and bending moment diagrams



Solution:

Free Body Diagram of Entire Beam:



$$\circlearrowleft\!+\ \sum M_B = -4C_y + 2(3) + 5(2) = 0$$

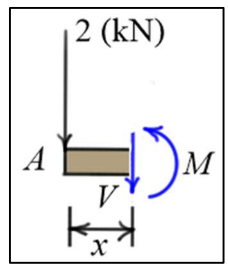
$$\Rightarrow \boxed{C_y = 4 \text{ (kN)} \uparrow}$$

$$\circlearrowleft\!+\ \sum M_C = 4B_y + 1(2) - 2(3) = 0$$

$$\Rightarrow \boxed{B_y = 1 \text{ (kN)} \uparrow}$$

Check: $\boxed{\sum F_y = B_y + C_y - 2 - 3 = 0}$ ✓

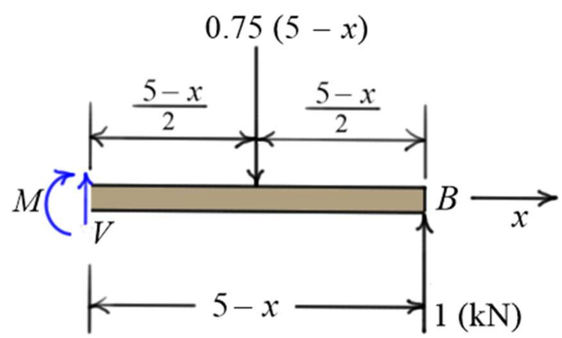
Free Body Diagram: $(0 \le x \le 1)$



$$+\downarrow \sum F_y = V + 2 = 0 \quad \Rightarrow \boxed{V = -2 \text{ (kN)}}$$

$$\circlearrowleft\!+\ \sum M_{cut} = M + x(2) = 0 \quad \Rightarrow \boxed{M(x) = -2x \text{ (kN-m)}}$$

Free body Diagram: $(1 \le x \le 5)$



$$+\uparrow \sum F_y = 1 + V - \tfrac{3}{4}(5-x) = 0$$

$$\Rightarrow \boxed{V(x) = \tfrac{3}{4}(5-x) - 1 \text{ (kN)}}$$

$$\circlearrowleft\!+\ \sum M_{cut} = 1(5-x) - \tfrac{3}{4}(5-x)\tfrac{1}{2}(5-x) - M = 0$$

$$\Rightarrow \boxed{M = (5-x) - \tfrac{3}{8}(5-x)^2 \text{ (kN-m)}}$$

Notes:

1. The ***shear force*** is ***constant*** from *A* to *C* and jumps (increases) by $C_y = 4$ (kN) at that point.
2. The ***shear force varies linearly*** in the range $(1 \le x \le 5)$. The slope is negative because $w(x)$ is a ***constant*** distributed load ***directed downward***.
3. The ***maximum shear force*** and ***bending moment*** both occur at the support at *C*.
4. The ***bending moment*** exhibits a ***local maximum*** at $x \approx 3.67$ (m) where the ***shear force*** is ***zero***. A zero shear force indicates a ***local maximum*** of the bending moment, but this ***may*** or ***may not*** be a ***global maximum***. In this case the ***global maximum*** is at the support at *C*.

5. The ***bending moment varies linearly*** in the range $(0 \le x \le 1)$ with a slope of $V = -2$ (kN).
6. The ***bending moment varies quadratically*** in the range $(1 \le x \le 5)$ starting with a ***positive slope*** and ***transitioning*** to a ***negative slope*** as indicated by the shear diagram.