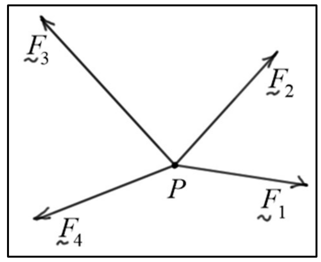
Elementary Statics
Equation Sheet #3: Particle Equilibrium, Cross Product, and Moments of Forces

Linear Spring Force: $F = k\,x$

Particle Equilibrium

$$\sum_i (F_x)_i = 0 \qquad \sum_i (F_y)_i = 0 \qquad \text{(2D)}$$

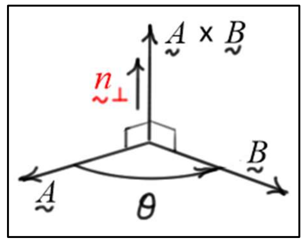
$$\sum_i (F_x)_i = 0 \qquad \sum_i (F_y)_i = 0 \qquad \sum_i (F_z)_i = 0 \qquad \text{(3D)}$$



Free Body Diagram

Cross Product

1. Geometric Definition: $\underset{\sim}{A} \times \underset{\sim}{B} = \left(|\underset{\sim}{A}|\,|\underset{\sim}{B}|\sin(\theta)\right)\underset{\sim}{n}_{\perp}$



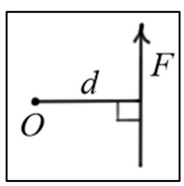
2. Calculation:

$$\underset{\sim}{A} \times \underset{\sim}{B} = \begin{vmatrix} \underset{\sim}{i} & \underset{\sim}{j} & \underset{\sim}{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \left(a_y b_z - a_z b_y\right)\underset{\sim}{i} - \left(a_x b_z - a_z b_x\right)\underset{\sim}{j} + \left(a_x b_y - a_y b_x\right)\underset{\sim}{k}$$

3. Properties of the Cross Product
   - Product is ***not*** commutative: $\underset{\sim}{A} \times \underset{\sim}{B} = -\left(\underset{\sim}{B} \times \underset{\sim}{A}\right)$
   - Product is distributive over addition: $\underset{\sim}{A} \times \left(\underset{\sim}{B} + \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \times \underset{\sim}{B}\right) + \left(\underset{\sim}{A} \times \underset{\sim}{C}\right)$
   - Multiplication by a scalar α : $\alpha\left(\underset{\sim}{A} \times \underset{\sim}{B}\right) = \left(\alpha \underset{\sim}{A}\right) \times \left(\alpha \underset{\sim}{B}\right)$

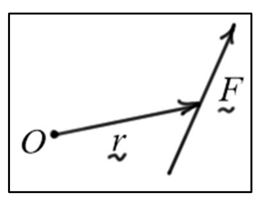
Moment of a Force

1. Scalar definition: $M_O = F\,d$



2. Vector definition: $\underset{\sim}{M}_O = \underset{\sim}{r} \times \underset{\sim}{F}$

3. Resultant Moment: $\left(\underset{\sim}{M}_O\right)_R = \sum_i \left(\underset{\sim}{M}_O\right)_i = \sum_i \left(\underset{\sim}{r}_i \times \underset{\sim}{F}_i\right)$



Moment of a Force about an Axis *a*

1. Scalar definition: $M_{\underset{\sim}{n}\text{-axis}} = \underset{\sim}{M}_O \cdot \underset{\sim}{n} = \left(\underset{\sim}{r} \times \underset{\sim}{F}\right) \cdot \underset{\sim}{n}$

2. Vector definition: $\underset{\sim}{M}_{\underset{\sim}{n}\text{-axis}} = \left(\underset{\sim}{M}_O \cdot \underset{\sim}{n}\right)\underset{\sim}{n}$